"G. H. von Wright's deontic Logic"

A critical Analysis of von Wright's Book "Norm and Action", and a Project for a more extended deontic Logic.

§ 1. The N-calculus.

G. H. von Wright's book "Norm and Action" deals with a deontic logic based upon a logic of transformations and a logic of actions. In the following text we will deal with an analogous deontic logic, but based upon a logic of transformations which has been changed in an important way.

G. H. von Wright introduced as basic elements for his logics, the variables p, q, r,... which should be understood as schematic representations of sentences expressing generic propositions and which describe generic states of affairs. One objection against this introduction might be made: the variables p, q, r... may only be introduced as basic elements, when states of affairs have an independent existence, and, that seems often rather doubtfull. Indeed, strong relations between states of affairs and processes can be shown. For instance: the sentence "The window is closed", does not describe an independent state of affairs, but a state of affairs being the case because some processes between the window and the closing mechanism keep it closed. Generally, say, the world is a p-world on a certain occasion. A transformation takes place and the world becomes a (-p)-world. Considering the fact that the state of affairs p is determined by some internal processes, there is an ambiguity concerning the state of affairs (-p). It can be determined by some other internal processes, or the former internal processes can keep working and transform the state of affairs (-p) again in the state of affairs p, from the moment the external causes of the transformation disappear. For instance: An inclined plane and a marble. Suppose there are two possible places where the marble can be at rest, up and down. When it's down, it stays down, forced by fysical powers. When someone puts it up and takes no further precautions, it will necessarely return to the first state of equilibrium.

G. H. von Wright's definition of the variables p, q, r,.. never takes the former considerations into account, but avoids the use of the notion of tendency [which is connected with the idea of logics as first proposed by Lucasiewiez].

Indeed, it isn't certain that the proposition "p tends to stay p" and "p tends to become -p" are mutually exclusive. In the next paragraph we shall try to construct a system expanding the possiblities of the logic of change and avoiding the difficulties of the notion of tendency. To handle the states of affairs we will introduce a new symbolism, by the following explicite definition:

pNp: This is the state of affairs characterized by the fact that the proposition p is true at the moment m_0 and stays true at the moment m_1, if every other proposition, relevant for the situation, is the same at the moments m_0 and m_1.

pN-p: This is the state of affairs characterized by the fact that p is true at the moment m_0 and -p becomes true at the moment m_1, if every other proposition, relevant for the situation, is the same at the moments m_0 and m_1.

-pN-p and *-pNp* can be defined in the same way. In this manner, one variable now corresponds to 4 possible states of affairs.

It is clear that the expression "the moment m_0" presumes the discontinuity of time. Although this is contradictory with the usual conception of time as a continuum, it seems easier to use the former conception. But, there is no objection to the idea that the moments m_0 and m_1 should be fictitious sections in the continuum of time. So, for use in the N-calculus, the two following definitions of time are possible:

1°. Time is represented as an infinite, linear class of isolated moments.

2°. Time is represented as a lineair continuum, and the moments are fictitious sections which are the boundaries of continous time-intervals.

Once time is introduced in our concept of states of affairs, an explicite notation of it seems necessarily. This notation can be defined as follows:

$(pNp)_{m_0}^{m_1}$: is the state of affairs pNp defined above, where m_0 is the former moment and m_1 the later one. But, m_0 and m_1 are not two "successive" moments, and the symbolism says nothing about the truth-value of the propositions p and -p, at every other moment, inside or outside the time-interval, than m_0 and m_1.

Now, we shall try to set up the axioms of the N-calculus.

1°.	-(pNp & pN-p)	(1)	-(pNp & -pNp)	(2)
	-(pNp & -pN-p)	(3)	-(pN-p & -pNp)	(4)
	-(pN-p & -pN-p)	(5)	-(-pNp & -pN-p)	(6)

Using the explicite notation of time axiom (1) should become

$$-((pNp)\,{}^{m_1}_{m_0}\ \&\ (pN\text{-}p)^{m_1}_{m_0}).$$

Although these axioms, should not be evidently at all, when we use the notion of tendency, it's clear that there is no possible state of affairs, where at the moment m_0 the proposition p is true, and where at the moment m_1, p and -p should be both true (axiom 1). Analogical evidences are valid for the other axioms.

2°. The four states of affairs are jointly exhaustive. When, at a given moment m_0, the world has the feature described by p, then at a later moment, it will have retained it, or lost it, or created it or not created it.

3°. We shall introduce now the notion of negation in our N-calculus, by means of -(pNp). The sole plausible explanation of this formula is the non-existence of pNp. This means, that one of the other three possible states of affairs has to be into existence. As a consequence, we have:

-(pNp) ↔ (pN-p) v (-pNp) v (-pN-p) (7)

The fact that the negation of a state-description is equivalent with the disjunction of three possible states of affairs is very important.

4°. Some distribution-axioms seem to be needed now in addition to the above principles. They are easily introduced as follows:

a. Consider first the case of one variable.

For instance: The state-description pN(pv-p) says that the feature of the world answers to the proposition p at the moment m_0, and that there is an incertitude about this feature at the moment m_1. So, in our vision we can say that we are not able to describe the real state of affairs :

pN(p v -p) ↔ (pNp) v (pN-p) (8)

Analogically to axiom (8) we can define:

(p v -p)N(p) ↔ (pNp) v (-pNp) (9)

So, the N-operator is disjunctive-distributive, relative to one variable, and even conjunctive-distributive. Indeed, (p & -p)N(p & -p) is an impossible state of affairs, and (pNp) & (pN-p)&(-pNp)&(-pN-p) an inconsistent N-expression, as shown by the axioms (1) to (6):

(p v -p)N(p v -p) ↔ pNp v pN-p v -pNp v -pN-p (10)

(p & -p)N(p & -p) ↔ pNp & pN-p & -pNp & -pN-p (11)

b. Consider now the case of two or more variables.

For instance: The state-description, (p & q)N(p v q) says that the feature of the world is (p & q) at the moment m_0, but that there is an incertitude about this feature at the moment m_1. Following the rules of the classic p-calculus, (p v q) can be replaced by

(p & q) v (p &-q) v (-p & q). So:
(p & q)N((p & q) v (-p & q) v (p & -q)). Analogically to axiom (8) we can replace this expression by:
(p & q)N(p & q) v (p & q)N(-p & q) v (p & q)N(p &-q).
Supposing that p and q are independent features of the world, the sole plausible meaning of an expression like (p & q)N(p & q) seems to be: (pNp)&(qNq). so :
(p & q)N(p v q) $\leftrightarrow$
(pNp)&(qNq) v (pNp)&(qN-q) v (pN-p)&(qNq) (12)
Analogical evidences lead to:
(p v q)N(-pv-q) $\leftrightarrow$
(pN-p)&(qN-q) v (pN-p)&(qNq) v (pNp)&(qN-q) v
(pN-p)&(-qN-q) v (pN-p)&(-qNq) v (pNp)&(-qN-q) v (13)
(-pN-p)&(qN-q) V (-pN-p)&(qNq) v (-pNp)&(qN-q).
So, the N-operator is:
a. disjunctive-distributive, following the axioms (10)-(12)
b. conjunctive-distributive, following the axioms (11)-(13).
The usefulness of the preceeding axioms in real life is easily shown by the following example. Consider axiom (8), when p says that a single atom x of a radioactive element is still in existence. Axiom (8) learns that the real state of affairs is unknown, indeed, nobody can say if the atom x will still exist at the moment m_1.
So it seems to me that the N-calculus can deal with the notion of probability, without further logical complications.

5°. N-expressions may be handled in accordance with the rules of the p-calculus, but with the restrictions (1-4).

6°. As already shown above, every state of affairs is a truth-function of elementary states of affairs. The truth-tables differ from "ordinary" truth-tables by some forbidden combinations:

1°. For instance, no two of the two-termed conjunctions of axiom (13) must be assigned the value "true." Generally, no two N-expressions concerning the same feature of the world must be assigned the value "true".
2°. Not all the possible N-expressions concerning the same feature of the world may be assigned the value "false".

§ 2. The T-calculus.

The N-calculus dealt with changes of the outlook of the world, caused by internal processes. The T-calculus which will be developped now, deals with changes due to external causes. It's clear that a change or transformation can only take place if there is an opportunity for it. So, the transformation of $(pNp)_{m_0}^{m_1}$ into $(-pNp)_{m'_0}^{m'_1}$ can only start at a moment when (pNp) is into existence, this means that m_0 has to be an earlier moment than m'_0. And m'_0 has to be not a later moment than m_1, because for moments later than m_1, there is an absolute incertitude about the existence of the opportunity. Such a transformation will be noted as follows:

(pNp)T(-pNp) or $((pNp)_{m_0}^{m_1}\ T\ (-pNp)_{m'_0}^{m'_1})$. Such expressions represent a class of transformations, the class which contents the transformations of the class of states of affairs pNp into the class of states of affairs -pNp.

When, starting from $(pNp)_{m_0}^{m_1}$, one wants to perform such a transformation that at the moment m'_0 (-p) becomes true, this can be done in the following ways:

$(pNp)_{m_0}^{m_1}\ T\ (-pNp)_{m'_0}^{m'_1}$ or $(pNp)_{m_0}^{m_1}\ T\ (-pN\text{-}p)_{m'_0}^{m'_1}$ or

$(pNp)_{m_0}^{m_1}\ T\ (pN\text{-}p)_{m''_0}^{m'_0}$.

Although they all answer to the question, these three transformations are quite different. So, 16 possible elementary transformations can be de fined:

(pN-p) T (-pNp)	(-pNp) T (pN-p)	(pNp) T (pN-p)
(pN-p) T (pNp)	(-pNp) T (-pN-p)	(pNp) T (-pN-p)
(pN-p) T (-pN-p)	(-pNp) T (pNp)	(pNp) T (-pNp)
(pN-p) T (pN-p)	(-pNp) T (-pNp)	(pNp) T (pNp)

(-pN-p) T (-pNp)
(-pN-p) T (pNp)
(-pN-p) T (-pN-p)
(-pN-p) T (pN-p)

G. H. von Wright's logic of change, which didn't deal with the N-calculus, contained only 4 elementary transformations :
pTp, pT-p, -pTp, -pT-p, which correspond respectively with (pNp)T(-pN-p), (pNp)T(pNp), (-pN-p)T(-pN-p), (-pN-p)T(pNp).

Let's now try to construct the axioms of the T-calculus.

1°. The 16 elementary transformations are mutually exclusive, but only when the time-intervals to the right of all T's are all the same (m'_0), and the time intervals to the left are all (m_0, m_1). This will always be so if time is not explicitly mentioned.

2°. The 16 elementary transformations are jointly exhaustive.
3°. We shall introduce now the notion of negation in our T-calculus, by means of -((pNp)T(pNp)). The sole plausible explanation of this formula is the non-happening of the transformation (pNp)T(pNp). This means that one of the other 15 transformations has to happen. Follows:
-((pNp)T(pNp)) ↔ the 15-termed disjunction of all possible elementary transformations, containing p as sole variable.
4°. (pNp)T(pNp)-expressions may be handled in accordance with the rules of the p-calculus, but taking (1, 2,3) into account. Brackets are used as in the p-calculus. Except in elementary transformations, which they make easier to read.
5°. Some distribution-axioms seem to be needed now in addition to the above principles. They can easily be introduced in the same way as the axioms of the N-calculus.
a. The T-operator is disjunctive-distributive.
b. The T-operator is conjunctive-distributive.
For instance:
((pNp)v(-pNp))T((-pN-p)) can be replaced by (pNp)T(-pN-p)) v ((-pNp) T(-pN-p)).
and:
(pNp)T(qNq) can be replaced by a 16-termed disjunction-sentence of two-termed conjunctions of transformations:
((pNp)T(pNp))&((qNq)T(qNq)) v ......
6°. So, every state-transformation is a truth-function of elementary state-transformations. The truth-tables differ from "ordinary" truth-tables by some forbidden combinations:

1°. No two of the T-expressions must be assigned the value "true."
2°. Not all T-expressions may be assigned the value "false".

Following the rules of the p-calculus, a T-expression is a T-contradiction (or T-tautology), when it expresses the contradiction (or the tautology) of its T-constituents.
7°. For the use of the truth-tables it is easier to construct first the positive normal form of the T-expressions. This is easily done as follows:
Using the distribution axioms pointed out above (5), the T-expression is written in its most expanded form. Then, using the definition of the negation of a T-expression, every negation is replaced by a disjunction of positive T-expressions.
Omitting now the T-contradictions (or T-expressions containing N-contradictions) we find the positive normal form of the T-expression.

8°. Some further definitions will be given, to make the following easier to understand:

State-description: A state-description is a conjunction-sentence of n-atomic pNp-expressions of n different atomic variables. For instance: pNp and pNp & qNq.
Change-description: A change-description is a conjunction-sentence of n-elementary T-expressions between pNp-expressions of n different variables p, q;..

For instance: (pNp)T(pNp) and ((pNp)T(pNp))&((qNq)T(qNq)). Consider n variables p, q, r... These variables determine 4^n different elementary state-descriptions. Every state-description determines 4^n state-transformations. Follows: n variables determine 4^{2n} or 2^{4n}different change-descriptions.

§ 3. **The d-calculus.**

The T-calculus dealt with changes of the states of affairs, caused by external factors. These external factors can be human beings or fysical powers. As pointed out in § 1, our purpose was the construction of a deontic logic based upon a logic of human actions. This logic, we shall try to develop now.

G. H. von Wright pointed out a difference between the meaning of an act and an activity. This difference is of no importance in our logic. Indeed, while von Wright accepted implicitly that m_0 and m'_0 were successive moments, they now are just fictitious sections of the time-continuum, which are not necessarely successive and discrete moments. So, the difference between act and ability decreases, just as the difference between processes and states of affairs has disappeared. An other difficulty appearing in von Wright's logic disappears too. There were two different ways of introducing the concept of trying to do something:

a. a logically incomplete mode of acting.
b. a logically incomplete mode of activity.

The difference between act and activity no longer existing, every ambiguity in the concept of trying to do disappears too. That our symbolism can cover both definitions is easily been by the following example of a logically incomplete mode of activity: An agent keeping open a self-closing door, or, an agent hanging on a cord above an abyss.

Let's try now to set up the symbolism needed for a logic of action:

1°. Accepting that an elementary transformation is the result of an elementary act, the correspondence is one to one, and the act can be represented by: d((pNp)T(-pN-p)) or $d((pNp)_{m_0}^{m_1}\ T\ (-pN-p))_{m'_0}^{m'_1}$. Such expressions represent a class of acts, the class which contents the acts of performing the transformations of the class (pNp)T(-pN-p).

2°. While the correspondence with the transformations is one to one, 16 elementary acts are possible. They can all be constructed by letting precede the symbol d each of the 16 possible elementary transformations.

3°. G. H. von Wright used the following symbolism: d(pTp). The p left of the symbol T stated the nature of the "opportunity". The condition of action had to be formulated in an additional restriction. For instance: "when p should have disappeared by itself". This additional restriction is no longer needed. Indeed, $(pNp)_{m_0}^{m_1}$ states the "opportunity" and the "condition of action": p being true at the moment m_0 states an opportunity to perform the act resulting in keeping it true at the moment m_0, or changing it into -p at the moment m'_0. But pNp says too, that p will not disappear by itself, at least till m_1, which is not earlier than m'_0, so pNp states a condition of action too. This condition of action only corresponds with von Wright's condition of action, and doesn't take other external influences into account. Some remarks about this kind of conditions of action will be pointed out later.

4°. I think no further explication needs to be given about most of the elementary acts. They represent acts describing the changing of a state of affairs, or the leaving a state of affairs unchanged. Some of them however require a little more explication:

$d((pNp)_{m_0}^{m_1}\ T(pN\text{-}p)_{m'_0}^{m'_1})$. This is the act transforming the state of affairs where p is true at m_0 and stays true at m'_0, but no longer at m'_1. It's interesting to remark that it's not necessary that -p should be true at m_1 to give the act some sense. Indeed, the external outlook of the world can still be p, while the internal processes characterizing pNp are changed.

$d((\text{-}pNp)_{m_0}^{m_1}\ T\ (pNp)_{m'_0}^{m'_1})$. The meaning of this act can be illustrated as follows: An agent lying in the water, is in such a state of affairs that he will sink by his own. The described act is pushing him under the water, and keep him there. Independently of the question if he could come to the surface by himself, one can do the preceding act. This illustrates clearly that actions like the ones just mentioned (absent or sen seless by von Wright)

have an important value when we consider human (moral) actions. For the analogical act d((pN-p)T(-pN-p)) the following example holds:
An agent commits suicide by hanging himself: (pN-p). Where p = the agent is living. Someone can prohibit this action: d(pN-p)T(-pN-p). This kind of act seems to hold for the problem of euthanasia too. When we take into consideration that m_0' cannot be a later moment than m_1, the actions just mentioned are never senseless, or nonsensical.

$d((\text{-}pNp)_{m_0}^{m_1}\ T(pN\text{-}p)_{m_0'}^{m_1'})$. If p should have been true at m_0' or not, if the preceeding act was not performed, has no importance. What we are changing are the internal processes characterizing the state of affairs. Analogical is d((pN-p)T(-pNp)).

The meaning of an act being introduced, we can try to state now the symbolism dealing with "forbearing". The comparison of d((pNp)T(pNp)) and von Wright's act d(pTp) shows that the actions are quite different. The action d(pTp) prevents that p should vanish. This was stated by the additional condition of action: pT-p. d(pTp) is thus rather to compare with d((pN-p)T(pNp)) and d((pN-p)T(-pNp)), than with d((pNp)T(pNp)). Indeed, these actions prevent that the feature p of the world should disappear. Still more important is, that von Wright had no possibilities to deal with the action expressed by d((pNp)T(pNp)), and neither with "forbearing," which he introduces as follows:

1°. Forbearing to perform the transformation pTp cannot be described by -(d(pTp)), because actions outside the human possibilities one does necessarely not, but nobody can forbear them.

2°. Neither can we define "forbearing" as the doing of non-changes because we introduced them as actions. For instance d(pTp).

3°. An agent, on a given occasion forbears the doing of a certain thing, if, and only if, he can do this thing, but does in fact not do it.

Retaining von Wright's definition, what's now the meaning of forbearing in our logic? First must be remarked that the result of forbearing is necessarily the non-happening of a certain change. Indeed, when an other agent let the change happen, we can no longer forbear it, because the opportunity for it has vanished. Say, f((-pN-p)T(pNp)). The result of this action (forbearing) is that the state of affairs pNp does not come into existence. But, then we must perform one of the following transformations: (-pN-p)T(-pN-p) or (-pN-p)T(-pNp) or (-pN-p)T(pN-p). So, every elementary f-action can be replaced by a disjunction-sentence of three elementary d-expressions. That means, that, in a formal way, we can forbear something we never can do, because forbearing only requires that we perform one of the other transformations. The reason why f and -d are not equal, even in our symbolism, is that -d should introduce other initial states of affairs, and we never can forbear an action when the necessary opportunity is not in existence. So, f can have some results, but they are not as distinct as the result of an action. Forbearing is a less distinct way of acting. For instance: What's the meaning of f ((pN-p)T(pNp))? We first put it in the positive normal form, which can be constructed in the same way as for the T-calculus. (the axioms needed for it will be introduced later). Follows: d((pN-p)T(-pNp)) v d((pN-p)T(-pN-p)) v d((pN-p)T(pN-p)). This is the disjunction of all, on a given occasion, possible actions, except the one we are forbearing.

In the following table I mention the final states of affairs instead of the action-results, which are transformations, because I want to show:

1°. The connection between initial states (opportunities and conditions of action) and final states.

2°. That the correlation between a transformation and the final state is not necessarely one to one.

The "elementary" f-expressions are mentioned too, although they are not elementary notions.

Condition of action Opportunity	Act or forbearance	Final states
pNp	d((pNp)T(pN p))	pNp
	f((pNp)T(pN p))	(pN-p)v(-pN-p)v(-pNp)
	d((pNp)T(-pN p))	-pNp
	f((pNp)T(-pN p))	(pN-p)v(pNp)v(-pN-p)
	d((pNp)T(-pN-p))	-pN-p
	f((pNp)T(-pN-p))	(pNp)v(pN-p)v(-pNp)
	d((pNp)T(p N-p))	pN-p
	f((pNp)T(pN-p))	(pNp)v(-pN-p)v(-pNp)
pN-p	d((pN-p)T(pN-p))	pN-p
	f((pN-p)T(pN-p))	(pNp)v(-pN-p)v(-pNp)
	d((pN-p)T(-pN-p))	-pN-p
	f((pN-p)T(-pN-p))	(pNp)v(-pNp)v(pN-p)
	d((pN-p)T(pN p))	pNp
	f((pN-p)T(pN p))	(-pN-p)v(-pNp)v(pN-p)
	d((pN-p)T(-pN p))	-pNp
	f((pN-p)T(-pN p))	(pNp)v(-pN-p)v(pN-p)

-pN-p	d((-pN-p)T(-pN-p))	-pN-p
	f((-pN-p)T(-pN-p))	(pNp)v(-pNp)v(pN-p)
	d((-pN-p)T(pN-p))	pN-p
	f((-pN-p)T(pN-p))	(pNp)v(-pN-p)v(-pNp)
	d((-pN-p)T(pN p))	pNp
	f((-pN-p)T(pN p))	(-pN-p)v(-pNp)v(pN-p)
	d((-pN-p)T(-pN p))	-pNp
	f((-pN-p(T(-pN p))	(pNp)v(-pN-p)v(pN-p)
-pNp	d((-pNp)T(-pN p))	-pNp
	f((-pNp)T(-pN p))	(pNp)v(-pN-p)v(pN-p)
	d((-pNp)T(pNp))	pNp
	f((-pNp)T(pN p))	(-pN-p)v(-pNp)v(pN-p)
	d((-pNp)T(-pN-p))	-pN-p
	f((-pNp)T(-pN-p))	(pNp)v(-pNp)v(pN-p)
	d((-pNp)T(pN-p))	pN-p
	f((-pNp)T(pN-p))	(pNp)v(-pN-p)v(-pNp)

An interesting consequence of the above discussion is the meaning of "ability" in our logic. von Wright defined: the ability to do something, and the ability to forbear the same thing are reciprocal abilities. This is no longer true in our system. Indeed, forbearing only requires the ability to do an other thing. (von Wright's remarks stay true for the failing to do). Further, the ability to do d((pNp)T(pNp)) doesn't assure the existence of the ability to do f((pNp)T(pNp)). Indeed, we are all able to leave the sun unchanged, but we are not able to perform a change of the sun. It will be shown that this formal ability to leave every state of affairs unchanged, when not prohibited by other agents, is of great importance.

Let's try now to set up the axioms of the d-calculus:

1°. The 16 elementary d-expressions, corresponding to the 16 elementary transformations, are mutually exclusive.

2°. The 16 elementary d-expressions are jointly exhaustive. Indeed, the four elementary states of affairs (pNp...) are jointly exhaustive with regard to the property p of the world. Starting from one state of affairs, one can only produce one of the three other states of affairs, and no more. So, the d-expressions are jointly exhaustive. This proves again the logical status of "non-elementary expressions" of the f-expressions. Moreover, the 16 f-expressions are included by the 16 elementary d-expressions. Indeed, forbearing or failing to do is the disjunction of three elementary d-expressions, which are mutually exclusive. So, failing to do is always equivalent with one d-expression. Which one, depends on the occasion.

Every df-expression expresses a truth-function of elementary d-expressions, because the operators d and f have some distributive properties, which are axiomatic in the df-calculus. They will be developped now:

a. The d-operator is disjunctively distributive in front of a change description.
   For instance: d(((pNp)T(-pN-p))v((qNq)T(-qN-q))) is equal to d((pNp)T(-pN-p)) v d((qNq)T(-qN-q)).
b. The d-operator is conjunctively distributive in front of change descriptions.
c. The f-operator is conjunctively distributive in front of a disjunction describing mutually exclusive transformations.
   For instance: f(((pNp)T(-pNp)) v ((pNp)T(-pN-p))) ↔ f((pNp)T (-pNp)) & f((pNp)T(-pN-p)). According to the definition of f, the later expression is equivalent with d(pNp)T(pNp) v d(pNp)T (pN-p). To find this result, replace the f-expressions by 3-termed disjunctions of d-expressions, use the axioms (a-b) and omit the inconsistent conjunctions. Follows:
   d(pNp)T(pNp) & d(pNp)T(pNp) v d(pNp)T(pN-p) & d(pNp)T(pN-p)
   It's easily seen that this expression is equal to the one formulated above, result which is perfectly in accord with the meaning of the "internal negation" of a d-and/or f-expression, notion we shall introduce later.
d. Some difficulties arise when we try to set up a distribution-axiom dealing with expressions like: f(((pNp)T(-pN-p))&((qNq)T(-qN-q))). According to von Wright this would mean: f((pNp)T(-pN-p)) & f((qNq)T(-qN-q)). But, according to this and other possibilities, we run into contradiction with our former definitions. The most plausible solution seems to be: Change f into d, and the conjunction into the 15-termed disjunction of all other conjunctions of possible change-descriptions, with pNp & qNq as initial state of affairs. According to the fact that the d-operator is conjunctively distributive, this results in a 15-termed disjunction of elementary d-expressions. This result is again perfectly in accord with the notion of "internal negation."
   Consider now the following f-expression, which has an inconsistent content: f((pNp)T(pNp) & (pNp)T(-pNp)). According to the preceeding definition, we can write it as a disjunction of consistent and

inconsistent d-expressions. By omitting the inconsistent ones, we get: d(pNp)T(pNp) v d(pNp)T(-pNp) v d(pNp)T(-pN-p) v d(pNp)T(pN-p). This is the disjunction of all possible actions with pNp as initial state of affairs. So, we see, that the f-expression itself is not senseless, although its content is inconsistent.

e. These four rules ensure the expression of every df-expression as a truth-function of elementary d-expressions. The rules for constructing truth-tables, tautologies and contradictions are analogical to the ones set up for the T-calculus. But, it seems easier to construct first the positive normal form. Indeed, then we must not set up special restrictions for df-expressions with an inconsistent content. Indeed elementary d-expressions never have an inconsistent content.

f. The construction of the positive normal form is easily done as follows:

Using the distribution axioms pointed out above, the df-expression is written in his most expanded form. (f-expressions must be replaced by the expressed d-expressions). Then, using the definition of the negation of a d-expression, every negation is replaced by a disjunction of positive d-expressions. (The negation of a d-expression we shall define immediately), omitting now the d-contradictions and the d-expressions occuring several times retaining only once, we find the positive normal form.

For instance: d((pNp)T(pNp)) v d((qNq)T(qNq)) can be resolved into a 96-termed disjunction of two-termed conjunctions of elementary d-expressions. Indeed, following the rules of the T-calculus, (pNp)T(pNp) can be replaced by (pNp)T(pNp) v (the 16-termed disjunction of all possible transformations with q as sole variable). Analogous for (qNq)T(qNq). Continuing as pointed out above, one will find the 96-termed disjunction.

g. Analogous to state- and change-descriptions, we shall define now the notion of an act-description. An act-description is a conjunction of n-elementary d-expressions with n different variables.

For instance: d((pNp)T(-pNp))&d((qNq)T(-qN-q)).

It's easily seen that n variables define atmost 4^{2n} act-descriptions. So, the positive normal form of a df-expression with n variables is a disjunction of O,... 4^{2n} conjunctions of n elementary d-expressions.A O-termed disjunction says that the df-expression is a df-contradiction. A 4^{2n}-termed disjunction says that it is a df-tautology. According to von Wright, we shall regard the positive normal form of a df-expression as consisting of segments answering to

the various conditions of application (initial states). Such a segment should be for instance:
d((pNp)T(-pN-p)) & d((qNq)T(-qN-q)) v
d((pNp)T(pN-p)) & d((qNq)T(qN-q)).

h. Finally, we shall discuss now the idea of negation of d-expressions. We shall distinguish between an external and an internal negation. The external negation is noted (-d), it is in this sense the notion of negation was used in the construction of the positive normal form of a df-expression. It is clear that the not-doing of a transformation, or a complex of transformations (in the positive normal form), requires the doing of one of all other transformations or complexes of transformations, answering the same variables. The external negation is thus quite analogical with the negation of a transformation. So, when a d-expression contains m members, the external negation necessarily has 4^{2n}-m members. Of cause, some or all of these members may vanish.
Consider now an expression in the positive normal form and divided into segments. Form the disjunction-sentence of all conjunction-sentences not-occuring in the segments, but answering to the same initial states as the conjunction-sentences in the segments. This disjunction is the internal negation of the d-expression.
For instance: d((pNp)T(pNp)) & d((qNq)T(qNq)) has a 15-termed disjunction of two termed conjunctions as internal negation. But this 15-termed disjunction is nothing else than f(((pNp)T(pNp))& ((qNq)T(qNq))). So, the internal negation of an act is the forbearing of the act. This shows the goodness of our axioms concerning the f-operator.

i. To make the following easier to understand, some further concepts will be introduced now:
   a. *external incompatibility*: Two acts will be said to be external incompatible, when the doing of the former requires the doing of the external negation of the later (by the same agent at the same occasion).
   b. *internal incompatibility*: Two acts will be said to be internal incompatible, when the doing of the former requires the doing of the internal negation of the later (by the same agent at the same occasion).

   It's clear that internal incompatibility requires external incompatibility, but not necessarily true is the reverse.
   c. *external consequences*: A d-expression is an external consequence of an other d-expression, when the implication containing the

later as antecedent, and the former as sequent, is a d-tautology.

d. *internal consequences*: A d-expression is an internal consequence of an other d-expression, when it is an external consequence and answers to the same initial state.

j. Very important for the following is the problem of extensionality of a logic. This problem will be discussed now.

If two or more d-expressions contain exactly the same variables, they are said to be uniform with regard to the variables. When expressions are not uniform they must be made uniform by a vacuous introduction of new variables. Before we yet have used this way of working, without discussing it.

Consider now for instance: A d-expression not containing p. It can be introduced by forming the conjunction-sentence of the given d-expression and d((pNp)T(pNp)) v -d((pNp)T(pNp)), or another disjunction-sentence of the same value.

So p can be introduced in a given T-expression by forming the conjunction-sentence of the given T-expression and (pNp)T(pNp) v -(pNp)T(pNp) or another disjunction-sentence of the same value. In a given state of affairs, p can be introduced by (pNp) v -(pNp), or another disjunction-sentence of the same value. Or even as follows: Say, qNq, then form (q&(pV-p))N(q&(pv-p)). In von Wright's symbolism d-expressions were not extensional with regard to p-or T-expressions. This followed from the special status of the f-expression and the notion "not able to do". In our symbolism this extension is possible. This is easily seen as follows:

Consider d((pNp)T(pNp)) (A^0)

p is tautologically equivalent with (p&q v p&-q). Follows that pNp is tautologically equivalent with (pNp) & ((qNq)v-(qNq)). Consider now: d(((pNp)&((qNq)v-(qNq)))T((pNp)&((qNq) v-(qNq)))). The change logic shows that the T-expression in the preceeding d-expression is tautologically equivalent with a 16-termed disjunction of two-termed conjunctions. Consider then:

d(((pNp)T (pNp))&(16-termed disjunction of two -termed conjunctions)). This expression is tautologically equivalent with: d((pNp) T(pNp))&(16-termed disjunction of elementary d-expressions with variable q)) (B^0). Consider now (A) and (B).

(A) says that a certain agent leaves pNp unchanged at a certain occasion. That means he does nothing.

(B) says that a certain agent leaves pNp unchanged at a certain occasion, and performs, at the same occasion, one of the 16 "jointly exhaustive" acts concerning the property q of the world.

In von Wright's system the later disjunction of d-expressions with variable q was not a tautology, because an f-expression was an elementary expression. In our system, our agent performs necessarily one of the 16 d-acts added to (A). The whole problem is then the meaning of d((qNq)T(qNq)), expression which has to cover the case, when the agent performs only (A) (and nothing else). It seems logically true, that an agent performing (A) performs always an infinite class of other acts of the type d((qNq)T(qNq)). Indeed, he leaves always an infinite class of aspects of the world unchanged.

The sole objection one can use is that when q is an aspect of the world the agent cannot change (on that occasion), even the 16-termed disjunction should be senseless. But as showed before, it is not senseless to say that every agent is always able to leave something completely unchanged, and what's more, performing d(pNp) T(pNp) doesn' trequire the ability to perform f(pNp)T(pNp). So, even that argument fails. The d-calculus is thus extensional with regard to p-, N-, or T-expressions.

§ 4. **The analysis of norms.**

Although the N-, T- and d-calculi are a sufficient basis for the construction of a deontic logic, a preliminary unquiry about the nature and the relations of norms seems to be needed. With this inquiry we will deal in the following notes.

A first relation, sufficiently pointed out by von Wright, will not be discussed here. It's the relation between commands and prohibitions, which can be defined as follows: "That which one ought to do, is that which must not be left undone"! Introducing the operator O, this kind of norms can be represented by the following expressions:

Od ((pNp)T(pNp)) says that the state of affairs pNp ought to be left unchanged or ought to be preserved when external causes should let it vanish.

Of ((pNp)T(pNp)) says that the state of affairs pNp must not be preserved when external causes should let it vanish, or ought to be changed.

Od((pNp)T(-pN-p)) says that the state of affairs pNp ought to be changed into the state of affairs (-pN-p).

Of ((pNp)T(-pN-p) says that the state of affairs pNp must not be changed into the state of affairs (-pN-p).

The other possible O-prescriptions (14 with elementary d-expressions, and 14 with "elementary f-expressions") are all alike. Perhaps it seems strange that we retain here the f-expressions, but as showed above, they are quite helpfull by explaining the notion of prohibition. It's clear now that commands and prohibitions are interdefinable, and that prohibitions are secondary with regard to commands. Moreover, prohibitions are less definite than commands, indeed a lot of actions always answer to a single prohibition. Consider for instance the absolute prohibition: "It's forbidden to perform any act"! This prohibition can be expressed by the command Od((pNp)T(pNp)), but never by a single prohibition. Sole the complex prohibition Of((pNp)T-(pNp)) says the same thing.

A second problem, not sufficiently pointed out by von Wright is the status of permissive norms. We will discuss it here in a quite other way as von Wright did.

1°. We shall try to give an explicite definition of "permission," without using common language. The reason therefore is very clear. Indeed, in Dutch, contrary to what happens in English, the ordinary way of expressing a permission is "*you may*", and the ordinary formulation of a prohibition is "*you may not*". So, one could say that the relation between a permissive norm and a prohibition is quite clear, in Dutch! Because this relation must not be dependent of one or another particular language, an explicite definition has to be constructed. This can be done as follows:
"To give a permission to do something, means, that a subject is allowed, by means of a permissive prescription, to choice between performing or not-performing one or more acts".
The question arises now if this definition is sufficient or not?

a. Shall we add to it: "to choice between forbearing or not-forbearing an act"?

No, because a permission to forbear an action is a permission to do one of the three acts, represented by the d-expressions in the disjunction-sentence which is equivalent with the f-content of the permission.

b. Shall we add to it: "taking care that no other agent prohibits the free choice of the subject"?

It is easily shown that the answer to this question has to be in the negative too. Indeed, when one gives an order to an agent, say Od(pNp)T(-pN-p), does he know then if no other agents will prohibit the ordered action? Of cause not, and when he wants to be sure of this, he has to give another complementary order. For instance:

"Every child, less than 14 years old, must go to school". This is the first order. The complementary order should be: "Everyone who retains a child from going to school, will be punished". But it is also possible that the authority does not give such complementary orders, so that an other agent can prohibit the fulfilment of the order, or it's even possible that there are agents who cann't be commanded by a certain authority, so that they are always free to prohibit the fulfilment of orders given by that authority. Concerning permissions we can argue in the same way. Indeed, a permission can be given alone or together with other prescriptions which insure a *free* choice. *For instance*:

A. "Every Belgian is free to choice his children's school".
(Some complementary prescriptions insure the free choice).

B. "I give you the permission to go out with your friend".
If this permission insures a free choice depends upon the decision of your friend. No complementary prescriptions insure the free choice.

There is no difference between these two forms of permissive prescriptions, only the complementary prescriptions insuring the free choice, are different. So, no extension of our explicit definition is necessary.

c. What's the meaning of "let the choice", expression occuring in our explicite definition?

To avoid discussion, we will give an explicite definition of this notion too: "Let the choice" stands for the following expression:

"The norm-subject can decide by himself, which of the actions, within the limits of the norm, he will perform, and which not. The norm-authority itself will not affect this decision."
So, a prescriptive permission is a declaration of "non-intervention" from the norm-authority.

2°. Now, we shall try to find out if commands and permissions are interdefinable. If yes, it must be possible to write down a permission by means of an O-expression (elementary or not). A systematic inquiry of this possibility can be done as follows:

a. "It is permitted to change, at will, the state of affairs pNp, or to leave it unchanged".

This norm is the most general permission one can give with regard to the state of affairs pNp. The only possible formulation of this norm, in terms of an O-expression, is a command with a tautological content, for that occasion: Od(pNp)T((pNp)v-(pNp)). (A)

The meaning of this norm is: "It is obliged to perform one of the four possible actions, but one is free to choice which one". These actions being jointly exhaustive, the norm (A) is equivalent with the given permission. One could argue that a norm with a tautological content is not a norm, but that kind of norms makes a certain state of affairs subject to norm, and so it has a certain value.

b. "It is permitted to change the state of affairs pNp into the state of affairs -pN-p".

This norm says that it is permitted to perform the transformation (pNp)T(-pN-p). While "let the choice" occured in our explicite definition, we know that the three other possibilities of transforming pNp, can not all be forbidden, and that no one of them may be obliged. What's now the real meaning of this kind of permission? Some examples will show it clearly:

A. When a teacher gives a pupil the permission to leave the class-room, because he has to catch his bus, the teacher gives a permission from which he knows how it will be used, although the pupil can still stay or leave.

B. A permission which is easy to formalize is the case, already mentioned, of a drowning person. The permission: "It is permitted to save a drowning-person", is a declaration of "non-intervention" from the norm-authority, when the subject performs the act of saving. Because the norm is a permission, one is free to choice, but this choice is not explicitly mentioned. So, one can not be sure that the norm-authority will not intervent, when one helps to drown the drowning person. And what's more, this intervention can be independent of the possible existence of a norm concerning the action of helping to drown. It is characteristic for this kind of norm that it says only something about the act which is the content of the norm, and nothing at all about the other possible acts.

Other permissive norms can perhaps be expressed in terms of commands, but it is never possible with this kind of "elliptic" norm. The previous analysis shows the vagueness of the notion of permission, even when defined explicitly. This vagueness makes every expression of it in terms of O-expressions impossible. So, P-norms will be retained as having an independent character. It is certainly not proved that permissions have an independent character, but it is proved that they can not be formalized in terms of O-expressions, as they occur in our symbolism. So, we finally are into accord with von Wright. But, where von Wright's decision was rather a contingent one, we showed the necessity of this decision, and moreover, we got a usefull explicite definition of "permission".

One thing has to be noted. In our explicite definition of the notion of permission occurs: "by means of a permissive prescription". There are no objections against this, as long as we handle only with prescriptions. But a generalization of our definition for norms which are not prescriptions must be preceeded by a control of the previous analysis.

In accordance to von Wright we shall retain: "Ought entails can" and "may entails can".

§ 5. Deontic Logic.

A. Categorical norms.

The deontic logic which we are going to develop now, shall only deal with "norm-kernels". This restriction is useful, in that sense, that such a logic can easily been generalized for all kind of norms, much easier for example, than a complete theory about prescriptions. Such a norm-kernel can clearly be defined as follows: "It's the part of a norm, containing the following components:

a. The normcharacter: this is an O- or P-character.
b. The norm-content: this are d-expressions.
c. The conditions of application: this are N-expressions".

The norm-kernels we will deal with in this first part, are normkernels of categorical norms. These are norms, with only one condition of application: the initial state of affairs or opportunity, expressed in the norm itself.

A first difficulty arising is that a deontic logic can only deal with descriptive norm-statements. But, the justification of the special laws governing this logic, must be layed down in the prescriptive use of the norms. So, a logical theory of prescriptive O and/or P-expressions must be constructed first. Although a lot of philosophical discussions were needed for it, I will only mention the results necessary for the further developpement of our logic.

1°. A selfconsistent norm is a norm with a consistent content. This means, that the d-expression following the letter O or P must be consistent, it must have a not-vanishing positive normal form. The special properties of the f-expressions are extremely important for the case. Indeed, as showed before, they are inconsistent in only one case, for instance: f((pNp)T(-pN-p)&(pNp)T(pN-p)&(pNp)T(pNp)&(pNp)T(-pNp).

2°. An inconsistent norm is a norm with an inconsistent content. This means, that the d-expression following the letter O or P must have a vanishing positive normal form.

3°. The only restriction for the consistence of p-, N-, and T- expressions was that they could meaningfull exist within their respective logics. The question if the self-consistence of OP-expressions can mean anything analogous to this, we tried to answer positively in the preceeding paragraph. In the following notes we will try to point it out clearly. First of all, we will give an explicite definition of the meaning of the negation of a norm in prescriptive use: "A norm is the negation-norm of another norm if, and only if, the two norms have an opposite character and their contents are the internal negations of each other". The consequences of this definition are quite different in von Wright's system and in the system here exposed. The importance of this difference we shall try to show clearly by the following examples:

Consider in von Wright's system the following norm: O(d(-pTp) v f(-pN-p)), his negation-norm was then (Pf(-pTp) v d(-pT-p)).

The analogous norm in our system is:
O(d(-pN-p)T(pNp) v f(-pN-p)T(-pN-p)), but the corresponding negation-norm is NOT P(f(-pN-p)T(pNp) v d(-pN-p) T(-pN-p)). Indeed, the internal negation of d(-pN-p)T(pNp) v f(-pN-p) T(-pN-p) is not f(-pN-p)T(pNp) v d(-pN-p)T(-pN-p), but d(-pN-p)T(-pN-p)!
The necessity of forming the positive normal form before constructing the internal negation of a d-expression, is showed very well by this example.

The explicite definition being the same as von Wright's, our system is in perfect agreement with his system, when the content of the norm-kernel is composed by d-expressions with different initial states of affairs. For instance:
Consider O(d(-pN-p)T(pNp) & d(-qN-q)T(qNq)). The negationnorm is then:

P(f(-pN-p)T(pNp) & f(-qN-q)T(qNq) v
f(-pN-p)T(pNp) & d(-qN-q)T(qNq) v
d(-pN-p)T(pNp) & f(-qN-q)T(qNq)).

The meaning of this norm can be defined, mutatis mutandis, in terms of von Wright: "The O-norm says that one ought to perform two definite actions, its negationnorm says that one may forbear at least one of them".

It is clear that with this definition of the negation of a norm, a norm and his negationnorm are not jointly exhaustive. The reason why nevertheless, we agreed with von Wright's definition is that, even when we should have taken Of v Pd v Pf as negation of Od (by analogy to the N- and the T- and d-calculi), the two expressions would not have been jointly exhaustive. So, that it was senseless to complicate the matter in that way.

4°. Selfconsistency of norms being defined, and disposing of the notion of negationnorm, we can start now an inquiry about the possibility of coexistence of norms. We shall first introduce the formal notion of compatibility of norms. This means the mutual consistence of two or more norms. A set consisting of such compatible norms, we shall call a consistent set of norms, taking into account, that a set with at least one inconsistent member is necessarely an inconsistent set too. We will discuss now successively the consistence of O-sets, P-sets and O + P-sets.

a. *O-sets*:

In accordance with von Wright we shall accept the following conclusion: "An O-set is consistent (the commands compatible) if, and only if, it's logically possible, under the given conditions of application, to obey all commands (collectively) which apply on that condition of application". But, some explanation seems to be needed for some special norms not occuring in von Wright's system. For instance: Consider the set of norms Od(pNp)T(-pN-p) and Of(pNp)T(pNp). Is this set consistent or not? Are his members compatible or not? The sole condition of application is pNp, and the corresponding parts, of the positive normal forms, answering to it, are: d(pNp)T(-pN-p) and d(pNp)T(pN-p) v d(pNp)T(-pN-p) v d(pNp)T (-pNp). The conjunction of these parts is consistent and the norms consequently compatible. But, the content of the second norm is an internal consequence of the first norm, so that, as we will see further, the first norm implicates the second one, and the set only consists of one norm.

b. *P-sets*:

Such sets are ipso facto consistent. Permissions never contradict each other. This is one of the basic logical differences between commands and permissions. Perhaps a restriction has to been made when norms of higher order are taken into account. For instance:

1. It is permitted to smoke.
2. It is permitted to prohibit smoking.

But, it's evident that this problem can not be solved by norm-kernels alone. Norm-authorities and subjects are important too in this case. Hypothetical norms (discussed further) will solve perhaps half of the problem. For instance: "It is permitted to smoke whenever nobody prohibits it". Such a norm would be an hypothetical norm with an additional condition of application: the non-existence of an other norm, given by an other authority. We will touch this problem again when we deal with hypothetical norms.

c. *O + P-sets*:

Such sets can always be divided into two sub-sets:

A. An O-set.

B. A P-set, which is always consistent.

So, we can accept the following conclusion:

"A set of commands and permissions is consistent (the norms compatible) if, and only if, it's logically possible, under any given condition of application, to obey *all* the commands collectively and to avail oneself of *each one* of the permissions individually, which apply on that condition.

Some general consequences of the preceeding discussion will be mentioned now:

A. A norm and his negation-norm are absolutely incompatible.

B. Two O-norms with contents which are each others internal negation, are absolutely incompatible.

C. Two norms, when not two P-norms, with contents which are internal incompatible, are absolutely incompatible.

D. A set of norms is incompatible when the contents of two of his members are internal incompatible. A P-norm is incompatible with a set of O-norms, when his content is internal incompatible with the content of one of the O-norms. The later incompatibility is not necessarily absolute.

E. Some explanation about the notions of absolute and mere incompatibility of norms seems to be needed here. This distinction is a consequence of the way of testing sets about there consistence. This inquiry has to be done as follows:

First consider the O-subset. Consider separately every condition of application, and the positive normal forms of the d-expressions occuring in the norms and answering to the different conditions of application. Then form the conjunctions of all the d-expressions answering to every condition of application separately. If every one of this conjunctions is inconsistent too, and the norms are *absolutely* incompatible. If only some of these conjunctions are inconsistent, the set is inconsistent too, but the norms are *not absolutely* incompatible. If none of these conjunctions is inconsistent, the set of norms is inconsistent and the norms are compatible. Then, perform the same test, for the O-subset and each P-norm separately. If every one of the so formed conjunctions is inconsistent, the O + P-set is inconsistent, and the P-norm *absolutely* incompatible with the O-subset, If only some of these conjunctions are inconsistent, the O + P -set is inconsistent, but the P-norm is *not absolutely* incompatible with the O-subset. If none of these conjunctions is inconsistent, the set is consistent too, and the P-norm compatible with the O-subset.

F. From the above explanation follows that mere incompatibility of the contents of two commands or a command and a permission does not entail, in our definition, the incompatibility of the norms. The incompatibility of the contents has to be internal. So, the explicite formulation of time can be of great importance and relevance. Indeed, the norms $O(d(pNp)_{m_0}^{m_1} T(pNp)_{m_0'}^{m_1'})$ and $O(f(pNp)_{m_2}^{m_3} T(pNp)_{m_2'}^{m_3'})$ are not incompatible, while the conditions of application are different. But they can be incompatible, when the time-intervals (m_0, m_1) and (m_2, m_3) have a common sub-interval. Analogically, it is useful to remember that the following norm is not selfinconsistent: $O(d(pNp)_{m_0}^{m_1} T (pN\text{-}p)_{m_0'}^{m_1'}$ & $d(pN\text{-}p)_{m_1'}^{m_2} T (\text{-}pN\text{-}p)_{m_1''}^{m_2'})$, but an obligation to perform two successive actions. So, we have to add to the above consquences: "Internal incompatibility of norms, and self-consistence of norms, not only requires identical conditions of application, but even the *same occasion in the time,* however this does not mean that we have to particularize the occasion.

G. Consider now the norms:

$$Od(\text{-}pN\text{-}p)_{m_0}^{m_1}\ T(pNp) \text{ and } Od(pNp)_{m_0}^{m_1}\ T(\text{-}pN\text{-}p).$$

These norms are not incompatible, in the above explained sense, because they have not the same conditions of application. Indeed, when the initial state is pNp, the first norm does not apply under the given circonstances. As long as the norms are given for one and the same time-interval, there are no difficulties at all, but when the commands are general with regard to the occasion, a Sisyphus-order comes into existence. Indeed, obeying the first command brings pNp into existence and the second order becomes applicable. And so on ad infinitum. A plausible formulation for such norms should be: $Od(\text{-}pN\text{-}p)_{n_i}^{m_i}T(pNp)_{n'_i}^{m'_i}\ \&\ d(pNp)_{n'_i}^{m'_i}\ T(\text{-}pN\text{-}p)_{n_i}^{m_i}$.

Where n_i, n'_i, m_i, m'_i are variables indicating intersections of the time-continuum, or variable time-moments. Certain is that such an order is not necessarely inconsistent, and even never inconsistent, when we make the restriction that the intervals (n_i, m_i) and (n'_i, m'_i) does not have a common sub-interval.

All norms mentioned here, while not internal incompatible, will be said to be external incompatible.

H. One more ambiguity is still to resolve. Indeed, when are Od(pNp)T(-pN-p) and Of(pNp)T(-pN-p) incompatible commands? "If, and only if, they handle about the same particular occasion, and when they are given by one and the same authority, to one and the same subject". When they are given by different authorities there is no logical contradiction, but a conflict of wills. So, the meaning of incompatibility of norms, or inconsistence of norm-sets, consequently must be restricted to incompatibility or inconsistence of a norm-set, prescribed by one and the same authority.

I. As pointed out in the preceeding discussions, self-inconsistent norms have a necessary non-existence. So, the question arises if some norms could have a necessary existence. Important for the case is of cause the idea of a tautological norm: "A norm will be tautological, if and only if, the positive normal form of the d-expression which is his content, contains all the act-descriptions answering to some or all of the conditions of application of the norm".

Such tautological O-norms does not oblige anything, and such P-norms does not permit anything, at least with regard to the conditions of application concerning the tautological part of the norm. Indeed, this was the case were O- and P-norms were interdefinable. While the content of the negation-norm of a tautological norm has a vanishing positive normal form, the negation-norm itself is a self-inconsistent norm and has a necessary non-existence. But, does this mean that the norm itself has a necessary existence? von Wright answers the question negatively, because:

a. We had to make the restriction that the state of affairs appearing within the norms, were subject to norm.

b. It is senseless to command something you will necessarely do, or to permit something you will necessarely do.

But, consider an agent confrontated with a situation which can be subject to norm. Is this situation the same, if there is no norm at all, or if there exists a tautological norm about it? The situation would be the same, if the normsystem within which the agent is living, should be closed by the norm: "Everything which is not explicitly ordered, forbidden or permitted is ipso facto permitted". But, such a norm does not exist in reality. And moreover, if the situation is not subject to norm, how can the agent know that one or an other logical consequence of other norms does not restrict his choice? A tautological norm would clear out this situation. So, that these norms are no longer senseless. But, so we showed that von Wright was right by pointing out that there are no forms with a necessary existence. And, what's more, a situation as just described can be subject to a retroactif norm, which will deal with the actions of the agent in that situation.

It is clear now, that a norm and his negation-norm are not jointly exhaustive, and that the fact that Od does not exist, does not entail that Pf in reality exists as a norm.

J. In the preceeding notes we touched an important question, it is to say, the notion of entailment between norms. Although I can not agree with von Wright, I will first mention his definitions, to point out the problem very clearly: "Consider a consistent set of selfconsistent norms and a selfconsistent norm. We want to determine the conditions of application under which the selfconsistent norm will be entailed by the set of norms. We add the negationnorm of the given norm to the set. This new set can be consistent, inconsistent or absolutely inconsistent. When it is absolutely inconsistent, the negationnorm is absolutely incompatible with the original set of norms. Then and only then, we say that the additional norm is entailed by the set of norms".

Let us test now the goodness of this definition in all possible cases:

a. Consider a norm-set and the norm Od(pNp)T(pNp). No difficulties arise.

b. Consider a norm-set and the norm Of(pNp)T(pNp). Say, that Pd(pNp) T(pNp) is absolutely incompatible with the set. This means that the act d(pNp)T(pNp) can not be performed, without disobeying at least one of the norms of the set. But, does this mean that one is obliged to do f(pNp)T(pNp) It is possible that the norm Od(pNp)T(-pN-p) is one of the norms of the set. And, Of(pNp)T(pNp) and Od(pNp)T(-pN-p) are compatible norms, but the former is entailed by the later, so that we may accept that Of(pNp)T(pNp) is entailed by the set.

c. Consider a norm-set and the norm Pd(pNp)T(pNp). Say that Of (pNp)T(pNp) is absolutely incompatible with the set. This means that no one of the acts expressed by f(pNp)T(pNp) can be performed, without disobeying at least one of the norms of the set. So, Pd(pNp)T(pNp) is entailed by the set. But, when Pf(pNp)T(pNp) is also absolute incompatible with the set, then Od(pNp)T(pNp) is also entailed by the set. Is it possible now, that one and the same set entails the norms Od and Pd? Or otherwise formulated: are Od and Pd compatible norms? We will answer the question later on.

d. Consider a set of norms and the norm Pf(pNp)T(pNp). Say that Od(pNp)T(pNp) is absolutely incompatible with the set. This means, that the act expressed by d(pNp)T(pNp) cannot be performed without disobeying at least one of the norms of the set. So Pf(pNp)T(pNp) is entailed by the set. But, the same difficulty as above arises, when, for instance, Od(pNp)T(-pN-p) belongs to the set of norms.

The above inquiry clearly shows that von Wright's definition of entailment is subject to discussion and criticism. The sole conclusion of this inquiry has to be: "A selfconsistent norm *can* be entailed by a consistent set of norms, if, and only if, his negationnorm is absolutely incompatible with the set of norms".

We will give now a new explicite definition, and we shall try to prove its goodness: "A self-consistent O-norm is entailed by a set of selfconsistent norms, if, and only if, his negationnorm is absolutely incompatible with the set. A selfconsistent P-norm is entailed by a consistent set of selfconsistent norms, if, and only if, his negationnorm is absolutely incompatible with the set and no one of the O-norms, with *identical* conditions of application, which make it impossible to avail oneself of the given permission, is a member, or is entailed by the set". Prescriptions which are entailed by a set of norms, we will call derived prescriptions, commands, prohibitions or permissions.

The derived norms of a set of norms are as much "willed" by the norm-authority as the original norms. They are in force, although they are not explicitly promulgated. To justify this step from the formal aspect to reality, von Wright says: "When a normauthority can not consistenly forbear an act, then it automaticcally has permitted the act". We add now: "when it has not ordered to perform it (or when it has not obliged it)". Indeed, it is evident that the following two norms, Od(pNp)T(pNp) and Of(pNp)T(-pN-p), which are logically compatible, can coexist in reality too, because they are both obligation-norms. The fact, that the former restricts the force of the later, is no objection for their coexistence. But the situation is quite different when we concern the norms Pd(pNp)T(pNp) and Od(pNp)T(pNp). Indeed, when we want to avail ourselves of a given permission, we must be able to make a "real" choice, and the O-orm prohibits this choice. So, we must say, that Pd(pNp)T(pNp) and Od (pNp)T(pNp) are incompatible norms. A set of norms which has Pd and Od as his members is an inconsistent set of norms. One could say that such a set can be consistent and maintained, because the members are "logically" compatible. But, such set should lead to irrational entailments. So, it is more safe not to maintain them. So, we will accept that something you are obliged to do, you never can be permitted to do, at least with regard to the same authority. Extreme caution is ordered now by dealing with the compatibility of norms. Indeed, the following set is NOW an inconsistent set of norms: Pd(pNp)T(pNp) and Of((pNp)T(-pNp) & (pNp)T(-pN-p) & (pNp)T(pN-p)). Indeed, the obligation entails the norm Od(pNp)T(pNp), and this norm is incompatible with the given permission.

I think, the preceeding restriction is sufficiently justified now. A lot of interesting results will follow, although the restriction seems at first somewhat contra-intuitive! *For instance*: Consider the norm P(d(pNp)T(pNp) v d(pNp)T(-pN-p)) and the norm Pd(pNp)T(pNp). It is evident that the later entails the former, as long as no other norms about pNp are in existence. But, when we consider the norm-set Pd(pNp)T(pNp) and O(d(pNp)T(pNp) v d(pNp)T(-pN-p)), the entailment does not hold anymore, although there are no LOGICAL objections. But, the O-norm has the same content as the entailed P-norm, and as said before, this two norms are incompatible. Still entailed is, for instance, the norm P(d(pNp)T(pNp) v d(pNp)T(pN-p)). Such entailments will be shown to be of great importance. Interesting for the case is also that the normset Of(pNp)T(pN-p) and P(d(pNp)T(pNp) v d(pNp)T(pN-p)) entails the P-norm Pd(pNp)T(pNp). Indeed, the negationnorm Of(pNp)T(pNp) is absolutely incompatible with the set of norms. Indeed, the conjunction of the contents of the members of the set is d(pNp)T(pNp), and the norm Od(pNp)T(pNp) is not entailed. So the entailment in question exists. The above results are all in good agreement with the results we layed down in the discussion about the idea of permission.

5°. Some distribution-rules with regard to norms seem to be needed now. We will mention them here as short as possible.

A. Consider the command: "Save him or let him drown"! Such a command lets the subject choice between two different acts. The formal expression of it is: O(d(pNp)T(pNp) v d(pNp)T(-pN-p)).

Does this order entail the commands: a. Od(pNp)T(pNp)
b. Od(pNp)T(-pN-p)?

Of cause not, indeed, the negationnorms of this two norms are not absolutely incompatible with the original norm. Furthermore, it is evident that the two norms together form an inconsistent set of norms, so they can not entail the original norm as a member. A distributionrule for this kind of "disjunctive obligations" can not be formulated.

B. Consider the command: "Open the door or let it open". Such an obligation is equivalent with a conjunction of two orders. *For instance*: O(d(pNp)T(pNp) v d(-pN-p)T(pNp)) entails the commands:

a. Od(pNp)T(pNp)
b. Od(-pN-p)T(pNp)

and these two norms together entail the original norm. This is the "rule of O-distribution", as formulated by von Wright.

C. A third form of obligation will be mentioned here, although the formalization can only be a mere suggestion. Consider: "Open the door and keep it open". Using the explicite definition of the time, we can write this command as: $O(d(pNp)_{m_0}^{m_1}T(-pNp)_{m'_0}^{m'_1} \;\&\; d(-pNp)_{m'_0}^{m'_1}T(-pNp)_{m_2}^{m_3})$. It is clear that other formalizations for this kind of norms are possible.

D. Consider now a permission, which content is a disjunction of acts which can be done under the same circonstances. Such a "disjunctive permission" can be equivalent with a set of permissions, but this is not necessarily so. Follows that no distribution-rule can be formulated.

E. Consider now a permission of the same form as the preceeding, but where the content is a disjunction of acts with different conditions of application. *For instance*: P(d(-pN-p)T(pNp) v d(pNp)T(-pN-p)). This norm entails the permissions:

a. Pd(-pN-p)T(pNp)
b. Pd(pNp)T(-pN-p)

and these norms entail together the original norm. This is the "rule of P-distribution" as mentioned by von Wright.

6°. Thanks to these rules of distribution, every prescription with several conditions of application may become "resolved" into a set of prescriptions, each one of which has only one condition of application. We will mention now some interesting calculations: Consider n variables (p, q, r,..). Then 4^n state-descriptions (pNp..) can be formed with them. To each one of these state-descriptions answer 4^n act-descriptions. $2^{(4^n)}-1$ disjunctions can be formed with these act-descriptions, when we count the act-descriptions themselves as 1-termed disjunctions. Each one can be the content of a P- or O- constituent, so that $2(2^{(4^n)}-1)$ constituents can be formed, but there where 4^n state-descriptions, so after all $4^n.2(2^{(4^n)}-1)$ or $2^{(2^{2n}+2n+1)} - 2^{2n+1}$ constituents are possible. For n = 1, the formula yields the value 120. But 8 of the constituents are tautological O- and P-norms, we wanted to mentain, but only once, and tautological O- and P-norms are equivalent. So, there are 120 - 4 = 116 different norm-constituents. To the general formula the term (-2^{2n}) must be added.

7°. Although the entailment of a norm by a norm-set has been rigourously discussed above, some additional notions about the entailment of norms will be given now:

A. If the content of a norm is an internal consequence of the content of an other norm, then the first norm is entailed by the second. But, with the restriction that the original norm and the entailed norm must have the same norm-character.

B. If the content of an obligation is an internal consequence of the conjunction of two or more obligations, then the first obligation is entailed by the set of obligations. This rule does *never* apply for a permission and a set of obligations. Indeed, the corresponding obligation is always entailed.

C. If the content of a permission is an internal consequence of the conjunction of two or more obligations and a permission, then the first permission is entailed by the set of norms, under the restriction that no obligation is entailed, or is a member of the set, which makes it impossible to avail oneself of the permission.

D. As several times mentioned with regard to incompatibility, these entailment-theorems are not valid for mere external consequences.

8°. Disposing now of a rather complete logical theory about prescriptive use of norms, we can set up the rules for testing OP-expressions in a truth-table. This kind of work deals of cause with descriptive use of norms, but is determinated by the preceeding theory about prescriptive use.

Consider an OP-expression. Replace the O- and/or P-constituents in it by the conjunctions of their constituents. The constituents must be made uniform with regard to all variables (p, q, r..) occuring in the entire OP-expression. The distribution of T(rue) and F(alse) over all POSSIBLE constituents, is subject to the following restrictions.

A. If the content of an O- or P-constituent is inconsistent, the constituent must be assigned the value F.

B. An O and a P-constituent with the same content can not be both assigned the value T. If an O-constituent makes it impossible to avail oneself of a permission expressed by a P-constituent, because the corresponding O-constituent is entailed, both constituents can not be assigned the value true or T.

C. If the contents of one or more O-constituents, of one or more O-constituents and one P-constituent, are internal incompatible, all of them can not be assigned the value T.

D. If the content of an O-constituent is an internal consequence of the content of an other O-constituent or of the conjunction of the contents of more O-constituents, then if the first is assigned the value T, all the others must be assigned the value T.

E. If the content of a P-constituent is an internal consequence of the content of one P-constituent and one or more O-constituents, then the former must be assigned the value true (T), if the later ones are all assigned the value T, and if the O-constituents do not entail the O-constituent corresponding with the original P-constituent.

Some kind of deontic tautologies, most of them appearing by von Wright too, will be mentioned now. Their test in truth tables is very easy.

Od(pNp)T(pNp) $\rightarrow$ -Pd(pNp)T(pNp) This is the above rule (B)
Of(pNp)T(pNp) $\rightarrow$ -Pf(pNp)T(pNp) rule (B)
Od(pNp)T(pNp) $\rightarrow$ -Of(pNp)T(pNp) rule (C)
Od(pNp)T(pNp) $\rightarrow$ -Pf(pNp)T(pNp) rule (C)
O(d(pNp)T(pNp) v d(-pNp)T(-pN-p) $\rightarrow$ Od(pNp)T(pNp)
$\rightarrow$ Od(-pNp)T(-pN-p)
Od(pNp)T(pNp) & Od(-pNp)T(-pN-p) $\rightarrow$ O(d(pNp)T(pNp) v d(-pNp)T(-pN-p))
P(d(pNp)T(pNp) v d(-pNp)T(-pN-p) $\rightarrow$ Pd(pNp)T(pNp)
$\rightarrow$ Pd(-pNp)T(-pN-p)
Pd(pNp)T(pNp) & Pd(-pNp)T(-pN-p) $\rightarrow$ P(d(pNp)T(pNp) v d(-pNp)T(-pN-p))
O(d(pNp)T(pNp) v d(-pNp)T(-pN-p)) & O(d(pNp)T(pNp) v d(-pN-p)T(pNp))
$\rightarrow$ Od(pNp)T(pNp)
O(d(pNp)T(pNp) v d(-pNp)T(-pN-p)) & P(d(pNp)T(pNp) v d(-pN-p)T(pNp))
$\rightarrow$ Pd(pNp)T(pNp)

This last tautology which appeared by von Wright, is not longer a tautology in our system. Indeed, the O-component entails Od(pNp)T(pNp), so, Pd(pNp)T(pNp) never can be entailed. But, the following tautology is still valid:

O(d(pNp)T(pNp) v d(pNp)T(-pN-p)) & P(d(pNp)T(pNp) v d(-pN-p)T (pNp))
→ Pd(pNp)T(pNp).

This follows, while the O-constituent is a disjunctive obligation.

B. Hypothetical norms.

Consider the following expressions:

a. If these and these contingences exist, then it is obligatory to act so and so.
b. It is obligatory, if these and these contingences exist, to act so and so.

The former expression is a descriptive sentence, it is an hypothetical norm-statement about a categorical norm.

The later expression can be seen as a descriptive or as a prescriptive sentence. If descriptive, it is a sentence saying that the whole expression (b) exists as a norm. That means it is a categorical normstatement. If prescriptive, it is the formulation of a norm and an additional condition. That means it is the formulation of an hypothetical norm. We will only deal here with the later kind of "if... then" clauses. The construction of a formal theory of such conditioned actions, requires the introduction of a new symbol: "/". Using this new symbol, we will first extend our logic of action.

An elementary "/-expression" we will call, an expression which is formed of an elementary d-expression to the left and the additional condition to the right of the symbol "/", expressing the "if ...then" clause. In von Wright's system, these additional conditions must be expressions of the same class as the normal conditions of applications, that means: transformations. Although we can not deal with all possibilities in this treatise, I will show that other expressions may be used for that purpose. Consider for instance:

a. d(pNp)T(pNp)/(pNp).

This /-expression is an elementary d-expression, because the additional condition is the condition of application itself.

b. d(pNp)T(pNp)/-pNp

This /-expression is inconsistent, because the additional condition and the condition of application of the d-expression can never coexist.

c. d(pNp)T(pNp)/(qNq)

This expression describes the act which lets the state of affairs pNp unchanged, when the state of affairs qNq is in existence.

The preceeding kinds of conditioning are very simular with von Wright's way of conditioning. The importance of the explicite notation of the time in such expressions we will discuss later on.

d. d(pNp)T(pNp)/(pNp)T(-pN-p)

Different interpretations are possible for such an /-expression. The following seems one of the most plausible to me: It is the description of the act which prevent that "pNp" should vanish, when, as a consequence of external influences (for instance acts of other agents, or changes of the other states of affairs which are in relation with pNp), a transformation should happen, which would transform pNp into -pN-p. The explicite notation is very relevant for this case. For instance: $d(pNp)_{m_0}^{m_1} T(pNp)_{m'_0}^{m'_1}$ $/(pNp)_{m_0}^{m_1} T(-pN-p)_{m'_0}^{m'_1}$. Indeed, the condition of action for such an act should be written (pNp) & ((pNp)T(-pN-p)). But such an expression makes no sense, because we do not know if pNp means the state of affairs before or after the transformation described by the d-expression. But using the explicite notion of the time, the following expression makes sense: $(pNp)_{m_0}^{m_1}$ & $((pNp)_{m_0}^{m_1} T(-pN-p)_{m'_0}^{m'_1})$. Indeed, the sole condition of action is then: $(pNp)_{m_0}^{m_1} T(-pN-p)_{m'_0}^{m'_1}$.

Remarkable too is the following expression, which looks apparently inconsistent: d(pNp)T(pNp)/(-pN-p)T(pNp). The inconsistence is evident when the conditions of application of both expressions cover the same time-interval. But interesting, and not inconsistent at all is the following expression: $d(pNp)_{m_0}^{m_1} T(-pN-p)_{m'_0}^{m'_1}$ / $(-pN-p)_{m'_2}^{m_2} T(pNp)_{m_0}^{m_1}$. This would express the act which changes pNp into -pN-p, whenever external influences have changed -pN-p into pNp. These apparently inconsistent /-expressions seem to be of the highest importance. Indeed, a lot of examples of this kind of actions can be found easily. For instance: An agent who has to open the locks, whenever the water stands to high.

e. $d(pNp)_{m_0}^{m_1} T(pNp)_{m'_0}^{m'_1}$ $/(qNq)_{m_0}^{m_1} T(-qN-q)_{m'_0}^{m'_1}$.

The meaning of this /-expression is nearly the same as the preceeding one, but with the conditions of application some more difficulties arise. Indeed, what is the meaning of $(pNp)_{m_0}^{m_1}$ & $((qNq)_{m_0}^{m_1} T(-qN-q)_{m'_0}^{m'_1})$?

The difficulty arising here is that the two conditions of application belong to different logics. Fortunately, the logic of change assures the following sense: (pNp)T((pNp)v -(pNp)) & (qNq)T(-qN-q). Indeed, when the world has the feature described by pNp, one of the for jointly exhaustive transformations, answering to pNp, must necessarely appear or happen.

f. d(pNp)T(pNp)/d(pNp)T(-pN-p)

g. d(pNp)T(pNp)/d(qNq)T(-qN-q)

Those two possibilities are quite analogical with the preceeding one, they have even the same conditions of application. If it were possible to subdivide, or specify with regard to the agents, the class of acts which let happen the transformations (qNq)T(-qN-q), the above kind of conditions would be very interesting. So, although our system is more differentiated than von Wright's system, a more expanded theory is still needed.

h. $d(pNp)_{m_0}^{m_1}\ T(pNp)_{m'_0}^{m'_1}/Od(pNp)_{m_0}^{m_1}\ T(pNp)_{m'_0}^{m'_1}$.

The meaning of this /-expression is clear and even trivial, it is the schematical expression which describes the act of obeying an order! d(pNp)T(pNp) /Od(pNp)T(-pN-p) would then represent the disobeying of an order. Such a /-expression, or such an act is therefor a priori forbidden!

i. d(pNp)T(pNp) / Od(qNq)T(qNq).

This represents the act which lets the state of affairs pNp unchanged, when there is an order to let qNq unchanged. It is evident that the subject of the additional condition must not "necessarely" be the same as the agent tho performs the act. But, it seems plausible to accept this.

j. d(pNp)T(pNp) / -Od(pNp)T(-pN-p).

This represents the act which lets pNp unchanged, when there is no order to change pNp into -pN-p/!

Although all possibilities mentioned above have some value, we will restrict ourselves to the use of transformations and state-descriptions as additional conditions of application. About the other possibilities we shall give only sporadic information. So the following definition of an elementary /-expression can be given: An elementary /-expression is an expression with an elementary d-expression to the left, and a elementary T-expression to the right of the symbol "/".

An atomic or molecular /-expression is an expression with an atomic or molecular d-expression to the left, and an atomic or a molecular T-expression to the right. It is clear that the additional condition can be written as a N-expression too. But, the use of mixed conditions will always be avoided, and whenever an incertitude about the occasion arises, we will use the explicite notation with regard to the time.

/-expressions are atomic /-expressions and molecular complexes of atomic /-expressions.

Important for the further development of our logic, is the fact that d-expressions always can be regarded as degenerated /-expressions, or limiting cases of /-expressions. This degeneration can be thought of in two ways:

a. The additional condition is the condition of application of the d-expression itself. For instance : d(pNp)T(-pN-p)/pNp. It is always possible to replace pNp by a 4-termed disjunction of T-expressions.

b. The additional condition is a tautological expression with regard to some other variables, than the ones appearing in the d-expressions. For instance: $d(pNp)^{m_1}_{m_0}$ $T(\text{-}pN\text{-}p)^{m'_1}_{m'_0}$ $/((qNq)^{m_1}_{m_0}$ v $\text{-}(qNq)^{m_1}_{m_0}))$. It is always possible to replace (qNq) v -(qNq) by the 16-termed disjunction of all possible transformations with conditions of application (qNq), (qN-q), (-qNq), (-qN-q).

An elementary /-expression can be constructed by putting one of the 16 types of elementary d-expressions to the left, and one of the 16 types of elementary T-expressions to the right of the symbol "/". So, $16 \times 16 = 256$ types of elementary /-expressions are possible. These 256 expressions are mutually exclusive, and form an exhaustive collection, because, in our convention, the 16 elementary types of d-expressions are "jointly exhaustive". As we mentioned before, some elementary /-expressions are self-inconsistent, when the same variables appear to the left and to the right of the symbol "/". When we suppose that the normal distribution of time is used, for instance in d(pNp)T(pNp)/(-pN-p)T(-pNp), then only 64 of the 256 elementary /-expressions are not self-inconsistent. This self-inconsistence does not hold necessarely, when we distribute time in another way.

For instance: $d(pNp)^{m_1}_{m_0}$ $T(pNp)^{m'_1}_{m'_0}$ $/(\text{-}pNp)^{m_3}_{m_2}$ $T(\text{-}pN\text{-}p)^{m'_3}_{m'_2}$, where m'_3 is an earlier moment than m_0. A practical example shows clearly that such a condition is not senseless. Consider for instance the proposition p: "The milk does not cook". Then, -p is the proposition: "The milk is cooking". A conditioned act could be for instance: "Let the milk which was cooking at m_0, continue to cook till m'_1 at least, when she should have been cooking at m_3, but was preserved from it by external influences." This example is even more interesting, because we must take into account the condition $(pNp)^{m_1}_{m_0}$ too, which requires an additional transformation, for instance: $(\text{-}pN\text{-}p)^{m_3}_{m'_2}$ $T(pNp)^{m_1}_{m_0}$. (other transformation are possible of cause).

We accepted that the expression d(pNp)T(pNp) would represent the degenerated expression of d(pNp)T(pNp)/(pNp). This supposition can not be proved mathematically, but it seems plausible, because (pNp) is the "necessary" and "satisfactory" condition of action while other ways of conditioning, for instance (pNp)T(-pN-p), are "satisfactory", but not necessary conditions of action. Conditioning by (-pN-p)T(pNp) even makes the expression self-inconsistent, when we use the conventional time-distribution.

Arises now the question of inconsistence of /-expressions. Two possibilities must be regarded:

a. A /-expression is inconsistent when the d-expression to the left of the symbol '/' is self-inconsistent.

b. A /-expression is inconsistent when the T-expression to the right of the symbol '/' is self-inconsistent.

Some restrictions must be added now with regard to the conditions of action. Indeed, the occasion for action must satisfy the additional conditions expressed by the T-expression and the conditions of application of the d-expression itself. So, when the conditions expressed by the d- and T-expressions are incompatible, although self-consistent, the [-expression becomes inconsistent. For instance: $d(pNp)^{m_1}_{m_0}\ T(pNp)^{m_1}_{m_0}\ /\ (-pNp)^{m_1}_{m_0}$ or $d(pNp)T(pNp)/(-pN-p)^{m_1}_{m_0}\ T\ (pN-p)^{m'_1}_{m'_0}$. But, such contradiction of conditions of application must always be expressed in the N-calculus, because the incompatibility is a consequence of the conditions of application of the d- and T-expressions. An expression of this contradiction in the T-calculus is not impossible, but could create confusion.

The preceeding can be generalized as follows: An atomic /-expression is inconsistent, when the conditions of application to the right (T- or N-expressions) are incompatible, in the N-calculus, with the conditions of application of the d-expressions to the left of "/". Follows, that an atomic /-expression is inconsistent when the conjunction of the two N-expressions is an N-contradiction! This formulation is the most general.

Sometimes it will be useful to mention all the conditions of application to the right of the symbol "/". This can easily be done, because d-expressions can be regarded as degenerated /-expressions. For instance: d(-pN-p)T(pNp) & f(-qNq)T(qNq)/(rNr)T(rNr) or d(-pN-p)T(pNp) & f(-qNq)T(qNq)/ ((-pN-p) & (-qNq)) & (rNr)T(rNr). This is the "longer form" of the /-expression. Follows the rule: A/-expression in the "longer" form is inconsistent if, and only if, the expression to the right of the symbol "/" is inconsistent.

The problem of uniformity of atomic /-expressions is easily resolved by adding to them the tautological expressions of the missing variables.

Some considerations about the distributivity of /-expressions will be pointed out now. Consider for this /-expressions where all the d-, T-, /- or N-expressions are in the positive normal form.

a. d(pNp)T(pNp)/((qNq)T(qNq) v (qNq)T(-qN-q)).

Following von Wright this expression should be equal to: (d(pNp)T(pNp)/(qNq)T(qNq))v(d(pNp)T(pNp)/(qNq)T(-qN-q)). It is evident that at a definite occasion pNp only one of the two transformations (qNq)T(qNq) and (qNq)T(-qN-q) can happen. So, for such a case the disjunction of acts is the only possible dissociation. (The conjunction would indeed represent a contradiction). But, if the expression is given for every occasion where pNp is the outlook of the world, this disjunction is certainly not identical with the original expression, because the disjunction signifies: "The act d(pNp)T(pNp) at the moments when (qNq)T(qNq) happens, or the act d(pNp)T(pNp) at the moments when (qNq)T(-qN-q) happens". (of cause performed by the same agent). A formalization of this general case must have the former case as a limiting case. So, the following formulation seems plausible:

$$((d(pNp)_{m_0}^{m_1}\ T(pNp)_{m'_0}^{m'_1}\ /(qNq)_{m_0}^{m_1}\ T(qNq)_{m'_0}^{m'_1})\ v$$
$$(d(pNp)_{m_0}^{m_1}\ T(pNp)_{m'_0}^{m'_1}\ /(qNq)_{m_0}^{m_1}\ T(-qN-q)_{m'_0}^{m'_1}))\ \&$$
$$((d(pNp)_{m_0}^{m_3}\ T(pNp)_{m'_0}^{m'_3}\ /(qNq)_{m_0}^{m_3}\ T(qNq)_{m'_0}^{m'_3})\ v$$
$$(d(pNp)_{m_0}^{m_3}\ T(pNp)_{m'_0}^{m'_3}\ /(qNq)_{m_0}^{m_3}\ T(-qN-q)_{m'_0}^{m'_3})).$$

In the limiting case when the time-intervals become equal, the two members of the conjunction become equal too, and only the disjunction remains!

b. d(pNp)T(pNp)/((qNq)T(qNq) & (rNr)T(rNr)).

This expression is equivalent with: (whatever happens to the distribution of time).

(d(pNp)T(pNp)/(qNq)T(qNq)) & (d(pNp)T(pNp)/(rNr)T(rNr).

c. (d(pNp)T(pNp) v d(pN-p)T(pNp))/(qNq)T(qNq).

This expression is equivalent with:

(d(pNp)T(pNp)/(qNq)T(qNq)) v (d(pN-p)T(pNp)/(qNq)T(qNq)).

Whatever happens to be the distribution of time.

d. (d(pNp)T(pNp) & d(qNq)T(qNq))/(rNr)T(rNr).

This expression is equivalent with:

(d(pNp)T(pNp)/(rNr)T(rNr)) & (d(qNq)T(qNq)/(rNr)T(rNr)), whatever happens to the distribution of time.

The preceeding distribution rules clearly show that every /-expression is a truth-function of elementary /-expressions. The way of putting /-expressions in the right form for testing them in a truth-table is quite equal with the way used for d-expressions! So, I will not discuss it here. It is also easily to show that categorical norms can be regarded as limiting cases of hypothetical norms.

The above exposed extended logic of action will be used now for the extension of the notion of an OP-expression. Only a few changes of the principles pointed out for categorical norms are necessarely:

a. An atomic OP-expression is an expression where O or P is followed by an atomic /-expression. The content of the norm is expressed by the d-expression to the left of the symbol „/"

b. The conditions of application of an hypothetical norm are the conditions of application of the d-expression to the left of the symbol "/", added, to the T- and or N-expressions to the right of the symbol "/".

c. The negationnorm of a norm is a norm with an opposite character, but with the same conditions of application, and the content of which is the internal negation of the content of the original norm. For instance: Od(pNp)T(pNp)/(qNq)T(qNq). The content of this norm is d(pNp)T(pNp), the conditions of application are (pNp)&((qNq)T(qNq)). The negation-norm is Pd(pNp)T(-pNp))/(qNq)T(qNq).

d. No difficulties arise by the extension of the preceeding for not atomic expressions. So, this is left to the reader. But, one very important difficulty arises, when we want to construct the negation-norm in some special cases. One of these cases can happen in von Wright's logic too. (but is not mentioned in Norm and Action). Indeed: O(d(pTp)/qTq v f(pTp)/(-qT-q)). The content of this norm is: d(pTp) v f(pTp). The conditions of application are: ((pTp) & (qTq)) v ((pT-p) & (-qT-q)). It is clear now that the internal negation of the content has a vanishing positive normal form. But it seems not senseless to suppose the existence of a negation-norm. The sole possible norm for the case is: P((f(pTp)/(qTq)) v (d(pTp)/(-qT-q))). When the original norm expresses that it is obliged to close the windows, when it rains, or to forbear to close them when it does not rain, the negation-norm says that it is permitted to close the window when it does not rain, or to forbear to close the window when it rains. This meaning seems evidently exact to me. The sole possibility to form this negation-norm requires the following additional definition: "When the content of an hypothetical normkernel is a disjunction of two or more d-expressions, with the same conditions of application, which appear in the norm containing /-expressions, where to the left of the symbol "/" the additional conditions are all the

same, we must form the negation of each member of the disjunction separately and add the corresponding conditions separetely too".
For instance:

O(d(pNp)T(pNp)/(qNq)T(qNq) v d(pNp)T(-pN-p)/(qN-q) T(-qN-q)). (1)
The content of this norm is: d(pNp)T(pNp) v d(pNp)T(-pN-p).

The conditions of application are: (pNp)&((qNq)T(qNq)v(qN-q)T(-qN-q)). It is easily seen that the internal negation of the content is d(pNp)T(-pNp) v d(pNp)T(pN-p). But, when we try to construct the negation-norm in that sense, we don't know how to distribute the additional conditions. Taking into account the preceeding definition, we form the negation-norm as follows:

P(f(pNp)T(pNp)/(qNq)T(qNq) v f(pNp)T(-pN-p)/(qN-q)T(-qN-q)). (2)
Consider now the norm:
O(d(pNp)T(pNp)/(qNq)T(qNq) v d(pN-p)T(-pN-p)/(qN-q)T(-qN-q)). (3)
the negation-norm of which is:
P(f(pNp)T(pNp)/(qNq)T(qNq) v f(pN-p)T(-pN-p)/(qN-q)T(-qN-q)). (4)

It is easily seen now, that case (1-2) is only a limiting case of the general case (3-4). To see that (2) is a limiting case of (4), it is sufficient to consider in (3) and (4) only ONE initial state of affairs pNp, and to put everything in the positive normal form. This extended definition being established we may accept the notions of compatibility and implication without change at all. Even the definition of consistence is the same as von Wright's: "The normkernel of an hypothetical norm is consistent if, and only if, the /-expressions following O- or P in the schematic formulation are consistent".

The following implications can easily be proved now:

a. Od(pNp)T(pNp) → O(d(pNp)T(pNp)/(qNq)T(qNq)).
b. Pd(pNp)T(pNp) → P(d(pNp)T(pNp)/(qNq)T(-qN-q)).

Indeed, it is evident, that something which is obliged or permitted unconditionally, is also obliged or permitted under certain conditions. The following implication is of cause a false one:

Od(pNp)T(pNp) → O(d(pNp)T(pNp)/(-pNp)T(pNp)).

In a still older system, von Wright accepted some tautologies, appearing in other deontic systems too. In his new system (Norm and Action), he rejected, them and tried to change them. We will consider these tautologies now, and try to see what they become in our system,

Formula I: O(A & B) ↔ OA & OB. The falsehood of this tautology (valid in the old system), is easily proved as follows:

Say: A = d(-pN-p)T(pNp) B = d(-qN-q)T(qNq). The formula becomes then: O(d(-pN-p)T(pNp) & d(-qN-q)T(qNq)) $\leftrightarrow$
Od(-pN-p)T(pNp)T(pNp) & Od(-qN-q)T(qNq).

This is not a deontic tautology at all. Indeed, if it were a deontic tautology, the norm O(d(-pN-p)T(pNp) & d(-qN-q)T(qNq)) should have to entail Od(-pN-p)T(pNp) and Od(-qN-q)T(qNq). This is not true, indeed the negation-norm of Od(-pN-p)T(pNp) is Pf(-pN-p)T(pNp), and this norm is not absolutely incompatible with the original norm. Indeed, the conditions of application, with regard to the variables p and q, of Pf(-pN-p)T(pNp) are: (-pN-p) & (qNq) v(-pN-p) & (-qNq) v(-pN-p) & (qN-q) v (-pN-p & (-qN-q). The sole condition of application of the original norm is:
(-pN-p) & (-qN-q).

With regard to the common condition of application, the two norms are incompatible, because the conjunction of there contents is inconsistent. But, the P-norm has more conditions of application then the original norm. So, the incompatibility is certainly not absolute and O(A & B) $\rightarrow$ OA & OB certainly does not represent a deontic tautology. But, we can replace it by the following deontic tautology:

O(d(-pN-p)T(pNp) & d(-qN-q)T(qNq)) $\rightarrow$
((Od(-pN-p)T(pNp)/(-qN-q)) & ((Od(-qN-q)T(qNq)/(-pN-p))). (Q)

So that a conjunctive categorical obligation is resolved into a conjunction of two hypothetical orders!

When we distribute time in a none conventional way, we find some interesting conclusions. Consider for instance the norm:

$$O(d(\text{-pN-p})_{m_0}^{m_1}\ T(\text{pNp})_{m'_0}^{m'_1}\ \&\ d(\text{-qN-q})_{m_2}^{m_3}\ T(\text{qNq})_{m'_2}^{m'_3}) \qquad (A.1)$$

where (m_0, m_1) is an earlier interval of time then (m_2, m_3).

this norm entails the norm $Od(\text{-pNp})_{m_0}^{m_1}\ T(\text{pNp})_{m'_0}^{m'_1}$. (A.2)

Indeed, the conditions of application are:

(A.1): $((\text{-pN-p})\ \&\ ((\text{qNq})\ v\ \text{-(qNq)}))_{m_0}^{m_1}\ \&$

$$((\text{-qN-q})\ \&\ ((\text{pNp})\ v\ \text{-(pNp)}))_{m_2}^{m_3}.$$

(A.2): $((\text{-pN-p})\ \&\ ((\text{qNq})\ v\ \text{-(qNq)}))_{m_0}^{m_1}.$

Indeed, it is evident that the outlook of the world during the later interval of time (m_2, m_3), can not give a condition of application for a norm valid during an earlier interval of time. That is the reason why the following norm is senseless:

$$Od(-pN-p)_{m_0}^{m_1} T(pNp)_{m'_0}^{m'_1} /(-qN-q)_{m_0}^{m_3}. \quad (B.1)$$

Follows, that the norm (A.2) has less conditions of application then the norm (A.1), and his negationnorm is absolutely incompatible with (A.1). So, the mentioned implication is tautologically true: ((A.1) → (A.2)).

But, the norm $Od(-qN-q)_{m_2}^{m_3} T(qNq)_{m'_2}^{m'_3}$. (A.3)

can not be implicated by (A. 1), because $((qNq) \vee -(qNq))_{m_0}^{m_1}$ is a condition of application of (A.3), but not of (A.1).

But, the following norm is again implicated by (A.1):

$$Od(-qN-q)_{m_2}^{m_3} T(qNq)_{m'_2}^{m'_3} /(-pN-p)_{m_0}^{m_1} T(pNp)_{m'_0}^{m'_1}, \quad (A.4)$$

because the negation-norm of (A.4) is incompatible with (A.1), and (A.4) has less conditions of application than (A.1). Indeed: $(-pN-p)_{m_0}^{m_1}$ is a condition of application of (A.1). The corresponding condition of application of (A.4) is: $(-pN-p)_{m_0}^{m_1} T(pNp)_{m'_0}^{m'_1}$.

And all other conditions are identically.

When we construct the norm (A.5), this is the norm (A.4), where the additional condition of application is replaced by $(-pN-p)_{m_0}^{m_1}$, this norm is implicated too by (A.1). Follows, that not-obeying the first part of the norm (A.1), does not allow to disobey the second part! It is easy now to verify that (A.2) and (A.4) or (A.5) collectively implicate the norm (A.1). Follows, that the following formula is a deontic tautology:

$$O(d(-pN-p)_{m_0}^{m_1} T(pNp)_{m'_0}^{m'_1} \& d(-qN-q)_{m_2}^{m_3} T(qNq)_{m'_2}^{m'_3}) \leftrightarrow$$
$$Od(-pN-p)_{m_0}^{m_1} T(pNp)_{m'_0}^{m'_1} \& Od(-qN-q)_{m_2}^{m_3} T(qNq)_{m'_2}^{m'_3} /(-pN-p)_{m_0}^{m_1} T(pNp)_{m'_0}^{m'_1} \quad (T)$$

So, we see, that when the time-intervals become identical, (B.1) is no longer senseless, and the tautology (T) changes into the tautology (Q) mentioned above.

Formula II: P(A v B) ↔ PA vPB. The falsehood of this tautology (valid in the old system), is easily proved as follows: It is possible that

P(A v B) exists unconditionally, but that PA and PB do not exist unconditionally. Indeed, the hypothetical normset Pd(pNp)T(pNp)/rNr and Pd(qNq)T(qNq)/-(rNr) implicates the categorical norm P(d(pNp)T(pNp) v d(qNq)T(qNq)). So, the above formula can not be valid, because if it was valid, a set of conditioned norm would insure the unconditioned existence of the same norms, and that is absurd. We will try now to find out if there is not a more general case where the formula should still be a deontic tautology. Consider for instance:

$$P(d(pNp)_{m_0}^{m_1}\ T(pNp)_{m_0'}^{m_1'}\ v\ d(qNq)_{m_2}^{m_3}\ T(qNq)_{m_2'}^{m_3'} \leftrightarrow$$

$$Pd(pNp)_{m_0}^{m_1}\ T(pNp)_{m_0'}^{m_1'}\ v\ Pd(qNq)_{m_2}^{m_3}\ T(qNq)_{m_2'}^{m_3'}.$$

This formula is obviously true. It is only when the time-intervals (m_0, m_1') and (m_2, m_3') intersect, that the formula has to be rejected and that the "identy" weakens to an implication.

Formula III: Sometimes, when an agent does something, he becomes "committed" to do something else. This means, when he does the first thing, he is obliged to do the second too. In von Wright's old system the formula of commitment was:
O(A → B). This means, it is obliged to do B if you do A. Two theorems concerning "commitment" were proved in that system too:
PA & O(A → B) → PB. This means, that the fact of doing something permitted, can only commit to a permitted act.
O(A → B) & O(-B) → O(-A). This means, that an act, the doing of which commits to do a forbidden act, is forbidden itself. We will try now to translate the notion of commitment in our logic.

a. O(A → B) was not only true for acts performed by one and the same occasion. So, we will discuss separetely the notions of "to commit oneself to an act on the same occasion", and "to commit oneself to an act on a later occasion." Consider the later of this possibilities. It leads to schematic expressions we only can denote here casually. Consider for instance the hypothetical norm:

Od(-pN-p)T(pNp)/(qNq) (A.1)

When on a certain occasion (-pN-p), the agent who is subject of the norm (A.1), performs an act producing qNq, he is obliged to obey the norm (A.1°). This is an "auto-commitment". When the state of affairs (qNq) has been produced by an other agent, we call it an Alio-commitment". A formalization of these possibilities can be proposed as follows:

$$\mathrm{Od}(\text{-pN-p})_{m_0}^{m_1}\ \mathrm{T}(\mathrm{pNp})_{m'_0}^{m'_1}\ /(\mathrm{qNq})_{m_0}^{m_1} \rightarrow$$

$$[\mathrm{d}((\text{-qN-q}) \vee \text{-}(\text{-qN-q}))_{m_2}^{m_3}\ \mathrm{T}(\mathrm{qNq})_{m_0}^{m_1}\ /(\text{-pN-p})_{m_0}^{m_1} \rightarrow \mathrm{Od}(\text{-pN-p})_{m_0}^{m_1}\ \mathrm{T}(\mathrm{pNp})_{m'_0}^{m'_1}]$$

The meaning of this formula is evident. The existence of the norm (A.1) implies that performing the conditioned act, creates at the time the categorical norm. But, it seems not so easy to find out, what will be the properties, the bearing and the possibilities of verification of such an expression. It is even not certain, that such an expression is not senseless in our calculus. Although we can adopt this expression to deal with "to commit oneself to an act on the same occasion" (this can easily be done by a conventional distribution of time), we will choice a safer way.

b. Consider the following formulae and say they are true:

pNp → qNq (B.1)
-(pNp) → -(qNq) v(qNq) (B.2)

Their meaning is obviously clear. Is it now possible that the meaning of an act of commitment is to produce a state of affairs as expressed by (B.1)? Then the formula would be of the following form:
d((pNp) & -(qNq))T((pNp) → (qNq)), when the initial state of affairs is (pNp) & -(qNq). Such an act can be performed by one of the following constituents:

d((pNp) & -(qNq))T((pNp) & (qNq))
d((pNp) & -(qNq))T(-(pNp)& (qNq))
d((pNp) & -(qNq))T(-(pNp)& -(qNq)).

The other possible initial states lead to analogical expressions, "to commit oneself to", would be than the obligation to perform one of these acts. For instance:
Od((pNp) & -(qNq)T((pNp) → (qNq)). This means; it is obliged to act in this way that (pNp) & -(qNq) is transformed into (pNp)&(qNq) or -(pNp)&(qNq) or -(pNp)& -(qNq). It is clear now, that when someone lets (pNp) unchanged, he is obliged to produce (qNq), too, or even to let it unchanged when existing.

The most general expression to perform the act, answering to the most general implication-act, would be:

Od(((pNp) v -(pNp)) & ((qNq) v -(qNq)))T(((pNp) & (qNq)) v
(-(pNp) & (qNq)) v (-(pNp) & -(qNq))). (B.3)

c. Suppose that the notion "to be obliged to produce qNq when you produce pNp" is only given for a -(pNp) & -(qNq) world. Then it is a prohibition to produce pNp when qNq is not produced at the same time. Follows:

O(d((-pN-p) v(-pNp) v(pN-p))T(pNp) & d(-(qNq))T(qNq) (Q.1)
v f(-(pNp))T(pNp) & d(-(qNq))T(qNq) (Q.2)
v f(-(pNp))T(pNp) & f(-(qNq))T(qNq)) (Q.3)

(Q.1) is a 9-termed disjunction of 2-termed conjunctions.
(Q. 2) is a 27-termed disjunction of 2-termed conjunctions.
(Q.3) is a 81-termed disjunction of 2-termed conjunctions.

Follows, that the norm (Q), has a consent composed by a 117-termed disjunction of 2-termed conjunctions of elementary d-expressions. Let us compare this norm (Q) with the norm (B.3). The content of (B.3) was composed, in his positive normal form, by a 208-termed disjunction of 2-termed conjunctions of elementary d-expressions. The difference between those results is easily explicable. Indeed, in the norm (Q), the initial states of affairs (pNp) and (qNq) are absent. And there are 91 possible conjunctions, where one or both of them represent the initial state of affairs. In von Wright's system, the formulae (B.3) and (Q) would have been irreconcilable, because the f-expressions had an independent status. In our system (B.3) and (Q) are equivalent, and the sole discussion is about the initial states pNp and qNq. It seems logically not to admit them as initial states, because when pNp was in existence, the norm would entail the obligation to produce qNq, when we let pNp unchanged, or the obligation to let pNp disappear when we do not produce qNq. Such an obligation is of a quite other character than the obligations mentioned here. We will not continu the discussion here, because it is of no relevance for the discussion of the two other important formulae of von Wright's old system. We shall accept the norm (Q) as the expression of "to commit oneself to", although it is not sure if it must not be extended with the initial states pNp and qNq.

d. PA & O(A → B) → PB. This formula is no longer valid in our system. But the first part of this norm, doesn't he imply in our calculus, an hypothetical norm? Translated in our calculus it becomes: Pd(-(pNp))T(pNp) & (the norm Q). This two norms are not incompatible. Indeed, the P-norm has more conditions of application than the O-norm, so that free choice is warented. It is easily seen, that the following norm is entailed by the consistent O + P -set.

$$\mathrm{Pd}(\text{-}(\mathrm{qNq})_{m_0}^{m_1}\ \mathrm{T}(\mathrm{qNq})_{m_0'}^{m_1'}\ /(\text{-}(\mathrm{pNp})_{m_0}^{m_1}\ \mathrm{T}(\mathrm{pNp})_{m_0'}^{m_1'} \qquad (C.1)$$

Indeed, this norm has only as much conditions of application as the (O + P)-set, his negation-norm is absolutely incompatible with the set and the corresponding O-norm is not entailed. Follows:

"An unconditioned permission to produce pNp, and an unconditioned prohibition to produce pNp and to let qNq unproduced, imply a permission to produce qNq when pNp is brought into existence by external influences".

The permission which is analogical to (C.1), but which has $-(pNp)_{m_0}^{m_1}$ as additional condition is entailed too. And this norm is even more interesting. Indeed, it is a permission to produce qNq from the moment that the conditions of application which are necessary to produce pNp, come into existence. It is interesting, that when pNp and qNq are admitted in the norm (Q) as initial states, and consequently in the categorical norm, von Wright's original formula is still valid. This is evident, because every initial state is a possibility to produce pNp or to let it unchanged.

e. O(A → B) & O(-B) → O(-A) is false too in our logic! She can be replaced by the following formula:
(The norm Q) & Of(-(qNq))T(qNq) → Of(-(pNp))T(pNp)/-(qNq). For which the same remarks as above can be made. Another interesting implication is:
(The norm Q)& Of(-(qNq))T(qNq) → Of(-(pNp))T(pNp)/(-(qNq)T(qNq)).
This prohibition, implicated by the O-set as well as by the O-norm implicated above, says, that the transformation (-(pNp))T(pNp) is forbidden, when qNq is not brought into existence by an other agent or by external influences. For instance:
"When someone is obliged to have a passport, and may not possess one without a stamp, which he himself is not permitted to print, he may not possess a passport when no other agent prints the stamp". The proofs of the above formula are easy and will not be mentioned here.

f. In the old system O(-A) → O(A → B) and
OB → O(A → B), were paradoxes which made the value of the notion of commitment quite debatable. This paradoxes still appear in our system. As von Wright in "Norm and Action", we need following extension of the notion of "commitment".
(the norm Q) & Pd(-(pNp))T(pNp)/-(qNq) & Pf(-(qNq)T(qNq)/-(pNp) (L)
Indeed, it is impossible to commit oneself to an act already commanded or forbidden. The two theorems (d) and (e) are still valid as theorems, but they do not deal with the notion of commitment. When we replace (Q) by the just proposed expressions, they decay to tautologies of the p-calculus. The implications we mentioned, where the additional condition was a transformation, do not decay, but stay useful as deontic tautologies. This is important. Indeed, in von Wright's new system no theorems about the notion of "commitment" were possible at all. In our system, some theorems are still valid deontic tautologies.

g. The formula (L) should better be replaced by a formula of the same form, but where the P-norms are replaced by expressions saying, that no norms at all, concerning the mentioned transformations, exist. But, this would complicate the matter, and so we will not discuss it here!

§ 6. **Post-face.**

The preceeding text deals only with the most important logically interesting paragraphs of my Dutch treatise about von Wright's "Norm and Action".

For both treatises I am indebted a lot to Prof. Leo Apostel from the university of Gent, who was always ready to pass constructive criticism.

Some changes appearing in the preceeding treatise are due to the kindly afforded critical observations of Prof. G. H. von Wright from the university of Helsinki.

Hunfred Schoeters