

JORDANUS DE NEMORE: A CASE STUDY ON 13TH CENTURY
MATHEMATICAL INNOVATION AND FAILURE IN CULTURAL
CONTEXT*

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Preliminary reflections on principles

As a philosophically minded historian of science, I am mainly interested in the thought-style of whole periods; in the changes of such styles over time; and in the reasons which can be given for their existence and development and for the differences between the scientific thought-styles of different cultures. This tunes me sympathetically to the idea of an evolutionary epistemology. Comparison with other evolutionary phenomena, particularly the phenomena of biological evolution, may suggest analogies and models which can be tried as possible explanatory devices even for the development of socially organized and epistemologically coherent knowledge; it may also warn us by showing how many pitfalls (and which sorts of pitfalls) turn up when one tries to make causal explanations of evolutionary processes (even when "cause" is taken in the wide, quasi-Aristotelian sense of "explaining reason").

On the other hand, as an empirically inclined historian I am also suspicious of the adoption of the analogy as a full-fledged *model* - an adoption which tempts us not to ask, e.g., *whether* the genotype-phenotype-distinction offers something valuable to the understanding of epistemological evolution but instead *what* are the analogies of these two levels. Furthermore, a tiny devil from my past as a physicist keeps whispering to me that when using water-waves as an analogy for acoustic phenomena one should not look for foam in the microphones. It may be a sound metaphysical principle that reality is one, and that homomorphous structures can be expected to turn up at different levels; but its soundness presupposes awareness that different levels are also *specific*, and that the relation between shared and specific structures can only be known *a posteriori*¹.

According to these reflections, the best thing I can offer to a

discussion of evolutionary epistemology is probably an investigation of a concrete historical case, locating a specific innovation of knowledge and its fate in context - "internal" as well as "external", with regard to a dichotomy which I find misleading. On an earlier occasion I have characterized the approach as "anthropology of mathematics" (since the investigation is in fact concerned with mathematics), using

... a term which suggested neither crushing of the socially and historically particular nor the oblivion of the search for possible more general structures: a term which neither implied that the history of mathematics was nothing but the gradual but unilinear discovery of ever-existing Platonic truths *nor* (which should perhaps be more emphasized in view of prevailing tendencies) a random walk between an infinity of possible systems of belief. A term, finally, which involved the importance of cross-cultural comparison.²

The case study is concerned with one of the few great mathematicians of the Latin Middle Ages, Jordanus de Nemore. The choice of this figure has the advantage for our present purpose that he is completely unknown as a person; our only access to him goes via his works. So, one is not tempted to replace the search for explanations by the writing of biography.

At the same time, the choice of the *opus* of a Medieval writer has its costs: The localization in the total directly relevant context requires that much material be brought together which is not otherwise found in one place or discipline (in order not to make things completely confusing I leave out the question of the indirectly relevant wider context, which would include the socio-economic background to the rise of the High Medieval University, the over-all social and epistemic organization of scholasticism, and the increasing "rationalization" of Medieval urban culture in general from the 11th century onwards³). If the whole analysis is not to stand as a collection of gratuitous postulates, quite a few details must therefore be included in the exposition (and only because of the existence of a more complete presentation have I avoided to make their number exorbitant - cf. below). On the other hand, the clustering of details may serve as a reminder that evolutionary argumentation must come to grips not only with empirically established processes but even with the difficulties to *establish* reliable empirical data pertinent to its inquiries.

The choice of a mathematical case involves advantages but also costs. Mathematics has always been considered a "hard

case" for "epistemologically relevant sociology of science" - be it "internal" or "external". So, an investigation which exemplifies *which* features of mathematical cognition can be explained by reference to social context is inherently interesting. Since, however, these features turn out to concern the *organization* of mathematical knowledge rather than concrete algorithms and formulae, experimental verifications and the final test of technological applicability will be absent; this puts some constraints on possible processes of "selective retention". The case presented in the following can therefore only be one specific case, demonstrating possible but not necessary structures.

The investigation was first presented in preliminary form to the XVI'th International Congress of the History of Science, Bucharest 1981, after which I put it aside unpublished because of other urgent work. In March 1985 I took it up again on occasion of the Ghent conference, and completed an extensive version which has recently been published ⁴ (with some revisions called for by the recent appearance of a critical edition of some central sources). The following exposition is a condensed version of this more extensive study of Jordanus and his environment⁵; it is followed by an epilogue in which I connect the results of the investigation to some general problems of evolutionary epistemology.

All translations of quotations into English are mine, unless the contrary is stated explicitly.

Jordanus de Nemore: Who, when, or what?

Since the mid-19th century, Jordanus de Nemore has been recognized as one of the greatest mathematicians of the Latin Middle Ages. So, Siegmund Günther (1887:156) characterized him as "second to only [Leonardo Fibonacci] and to no other aiming toward the same goal", while Moritz Cantor (1900:53) considers the two together "the boundary posts of a new era for the mathematical sciences". No wonder that considerable amounts of effort and speculation have been dedicated to the identification of an author of such scientific quality.

Efforts expended in vain: Evidence believed to connect Jordanus to Toulouse University turns out to be a red herring, and the identification with Jordanus de Saxonia appears to build on loose rumours. No single source known to date informs us about Jordanus as a person. We are left with references to his works, and with these works themselves.

It is evident from the works that Jordanus wrote in the Latin

scholarly environment; we know that he wrote before the mid-13th century, when his major works turn up in the *Biblionomia*, a catalogue which Richard de Fournival, writer, physician and chancellor at the Amiens Cathedral "well versed in mathematics", drew up of his extensive private library at some moment between 1243 and 1260. A similar time-limit is furnished by a reference to Jordanus' *Arithmetica* and by extensive use of the same work in Campanus de Novara's version of the *Elements*, written presumably between 1255 and 1259.

As to the *terminus a quo*, Jordanus makes use of a wide variety of 12th century translations, some of which date from its later part; hence it is improbable that he can have worked before the early 13th century when a reasonable diffusion of manuscripts had taken place, and impossible that he can have worked before c. 1175.

So, Jordanus worked during a critical period of the history of European learning: The period when Latin Europe had to digest the rich meal of translations made during the 12th century, and to re-create its own system of learning, socially and intellectually, on a higher level. It is the period when the schools and guilds of *magistri* and *scholares* of Bologna, Paris and Oxford developed into universities, and when the first universities created *as such* came into being in Cambridge, Padua, Toulouse and various other places. It began by the spread of Aristotelian philosophies tainted by Neo-Platonism and Avicennism, and it ended by the accomplishments of Saint Thomas and Albert the Great. It was marked by a series of processes and prohibitions, beginning by the condemnation of Amaury of Bène and David of Dinant and of the Aristotelian *Libri naturales* in Paris in 1210. Later, in 1215 and 1231, the teaching of Aristotle's natural philosophy was again forbidden in Paris, and in 1229 Pope Gregory IX warned the Paris theologians not to adulterate the Divine Word by the admixture of profane philosophy⁷. In 1228, the statutes of the recently founded Dominican order - an order to which *study* was a central activity - forbade the reading of pagan philosophers and of secular arts in general, including the Liberal Arts, reviving the old Augustinian fear of secular intellectual curiosity⁸. And yet, already in 1231 the prohibition of the *libri naturales* was mitigated by a promise and an attempt to make a censured and "inoffensive" edition, and when in c. 1250 Albert the Great wrote the first part of his huge paraphrase of Aristotle, beginning with *Physica*, the "book on nature" *par excellence*, it happened according to his own words on the request of the "fratribus ordinis nostris" - i.e. on the request of Dominicans in need of a book from which they could under-

stand Aristotle's natural philosophy⁹.

So, if we cannot know his father, his mother, his genealogy or just his nationality, we may approach Jordanus and the "Jordanian question" from another angle, more interesting in fact from the points of view of Medieval cultural history and the dynamics of the evolution of knowledge. We may ask how Jordanus fitted into the world of learning inside which he lived, *to judge from his works*.

The "Latin quadrivium"

Jordanus was a mathematician, and so we shall have to take a closer look at the development of Medieval mathematics up to the late 12th century.

The Early and Central Middle Ages has inherited from Antiquity the scheme of the Seven Liberal Arts: Grammar, rhetorics, dialectics (those three *artes sermonicales* constituting the *trivium*), and the four mathematical arts arithmetic, geometry, music, and astronomy (the *quadrivium*). Take as witness that most cited authority of the Middle Ages next to the Bible: Isidore of Seville. Book I of his *Etymologies* deals with grammar, book II with rhetorics and dialectics, book III with the quadrivial arts.

The level of mathematical knowledge as presented by Isidore in the early 7th century is low - no lower, perhaps, than the mathematics of the average Roman gentleman who had been taught in *his* times the Liberal Arts, but significant in view of the fact that Isidore was probably the most learned man of his century in Latin Europe.

So, Isidor can hardly be considered a transmitter of mathematical *knowledge*. But he did transmit. He transmitted the conviction that mathematics was a legitimate and an important part of Christian culture. So *Etymologies* III, iv, 1:

Knowledge of numbers should not be held in contempt: In many passages of the Holy Scripture it elucidates the mysteries therein contained. Not in vain was it said in the praise of God: *You made everything in measure, in number, and in weight.*

And III, iv, 3:

By number, we are not confounded but instructed. Take away number from everything, and everything perishes. Deprive the world of computation, and it will be seized by

total blind ignorance, nor can be indistinguished from the other animals the one who does not know how to calculate¹⁰.

So, in the 7th century "mathematics" was mainly a shell of recognition containing almost no substance. Numbers were less the subject of arithmetic than that of sacred numerology. Only Bede's writings on *computus*, i.e., sacred calendar-reckoning, gave to the early 8th century the beginnings of a new tradition for elementary applied mathematics which later met with great success.

The early "Carolingian Renaissance" meant little for filling the empty shell, except that Bede's writings were spread on the Continent. The shell itself, however, was strengthened - that is, the recognition of mathematics as an important subject was enhanced by the spread of Martianus Capella's *Marriage of Philology and Mercury* and of Cassiodorus' *Institutiones*. Due, however, to this same recognition those mathematical works which were discovered were increasingly copied at the *scriptoria* of the reformed Carolingian monasteries in the 9th century. So, Boethius' *Arithmetic* began to circulate in a certain number of copies, and various geometrical writings were discovered and spread: Excerpts from Boethius' translation of the *Elements*, and various extracts from Roman agrimensor-writings (which were apparently used as a basis for Liberal Arts geometry rather than as surveyors' manuals). As, in the later 10th century, monastic and cathedral school teaching expanded, this came to serve.

For one thing, teaching of Boethian arithmetic by means of a board game *rithmomachia* was practiced at least from c. 970 onwards; Boethian arithmetic must hence have been studied systematically in certain monastic schools from that time on.

Next, and more significant, a new abacus was invented, apparently under some inspiration from the Islamic world¹¹. A number of treatises describing it date from the late 10th and the early 11th century. It combined with the agrimensor- and "Boethian geometry"-traditions, and so became part of quadrivial *geometry*, paradoxical as this may seem. It had intercourse with the *computus*, too.

Finally, we have direct testimony of an awakening mathematical interest by the turn of the century, both as concerns Gerbert's teaching in the cathedral school at Rheims (organized according to the quadrivial scheme) and a number of letters exchanged between various scholars.

The general growth of intellectual interests at the cathedral schools during the 11th and early 12th century had to a large

extend the form of interest in the Liberal Arts. Truly, theology, medicine, and Canon Law were estimated, but in practice carried by no educational program comparable to that of the basic Liberal Arts. And so, the flourishing of the "12th century Renaissance" was seen by contemporaries as the apotheosis of these Arts - as can be seen in all sorts of sources, from the Royal Portal of the Chartres Cathedral to humble treatises on the Abacus.

So, by the early 12th century, the Isidorean shell of recognition had been filled out by what we might call the *Latin quadrivium*, i.e., a quadrivium founded almost exclusively on the tradition of Latin Christian learning and on the innovations which had been introduced homeopathically inside the Latin Christian world: Boethian arithmetic; the "sub-Euclidean" agrimensorgeometry including Euclidean fragments, the abacus and the figurate numbers; music, as dealt with in Boethius' *De musica*; and astronomy including elementary theory of the celestial sphere, *computus*, some elementary treatises on the astrolabe, and some astrological *opuscula* which had made their way into Latin learning from the late 10th century onwards, primarily through the contact in Lorraine. It is an open question how much of this was learnt by normal students - the quadrivium was not *à la mode* as was dialectics¹²; but much of it was *taught* in a number of schools, and some fragments and outlines were remembered by former students. In this way, the "Latin quadrivium" became an integrated part of the cultural traditions of the High Medieval scholarly environment.

A few distinctive qualities of the Latin quadrivium deserve to be listed. Firstly that *arithmetic* had the absolute primacy as regards prestige. This may owe much to the circumstance that Boethius' *Arithmetic* was the only really coherent treatise at hand, for all its lack of mathematical substance; it may also reflect a connection between a slow and gradual but indubitable calculatory rationalization of aspects of social practice¹³ and the growing interest in mathematics.

Secondly, the quadrivium was a *not quite autonomous part* of the global system of Liberal Arts, as it is well illustrated in the didactical introductions to Boethius' *Arithmetic*: They are stuffed with references to the verbal arts of the trivium, especially to dialectics.

A final characteristic of the Latin quadrivium was that it was *recognized not to be complete*. From its own basic authorities (Boethius, Cassiodorus, Martianus Capella, Isidore) it was known that a number of inaccessible Greek authors had laid its foundations. Pythagoras, Archytas, Euclid, Archimedes, Apollonios,

Nicomachos and Ptolemy are mentioned along with a number of others, and of these only Nicomachos was known through translations.

“Christian” quadrivium and “Christian” learning

That this flaw of the Latin quadrivium was really felt like one by those interested in quadrivial matters is seen in the heroic story of the 12th century wave of translations - a wave of which it was a motive force. I shall only quote a 14th century biographical note on Gherardo of Cremona, the greatest 12th century translator: “Educated from the cradle in the bosom of philosophy”, he became dissatisfied by the limits of Latin studies and “set out for Toledo” to get hold of the *Almagest*: he stayed there translating the Arabic treasures “until the end of life” (the whole impressive note is in Boncompagni 1851:387f).

By the mid-12th century, translations of the *Elements* began to circulate, and some treatises ascribed to Ptolemy were in use. When soon afterwards Gherardo’s translation of the *Almagest* became available, Latin Europe had at long last got a complete, “Christian” quadrivium - the term “Christian” taken without its religious connotations, as “belonging legitimately to Christian culture, to its background in Antiquity, and to its school curriculum”, - i.e. taken as an ethno-cultural characterization.

If we look at the non-mathematical branches of 12th century “Western” culture, there are good reasons to use the word in this sense (although, of course, men of the 12th century never distinguished between a religious and a cultural Christianity). For one thing, pagan Classical Antiquity was the ever-present background in all branches of scholarly activity, - and it was often more than background.

Not only, however, were there reasons to see Antiquity as belonging to one’s own circle, “les gens d’ici”. There was also a tendency to see Islam as a distinct, strange and intimidating world - “les singes d’à côté”. This did not only regard faith, culture, morals, and political and ecclesiastical power. Even the modern stereotypes of the Healthy West and the Unhealthy and Venomous East find their parallel in the 12th century.

In general it is not easy to find such antagonism expressed directly in writings concerned with learning. The great 12th century translators seem to have been just as eager to transmit Islamic or Medieval Jewish as Greek authors. Enthusiasts as well as antagonists of the new, imported learning were, so it looks, just as enthusiastic about genuine Islamic learning as about the

full Greek heritage, or just as much against Galen and Euclid as against any modern pagan philosophy.

Still, at closer inspection much of the evidence suggests a hidden affinity for the Greek part of the new learning. This is revealed if we ask the questions, *who* among the many translated authorities became favourites, and *for which purpose* they were used.

Two things become clear as soon as these questions are posed. Those authors who belonged to the Ancient heritage became the main authorities, and the main threat according to those suspicious of the new learning; further, Muslim authorities were most often not used independently but rather as support or commentary in connection with questions or works correctly or falsely ascribed to Antiquity. There are also certain indications that Ancient and Jewish authorities were considered either more reliable or less dangerous to cite than Muslims.

A comparison of the different styles of translations from the Greek and from the Arabic leads to similar conclusions. Arabic texts, even when translations from the Greek, were treated as *objects to be used*, and not always very formally. Greek texts, on the other hand, were sacred objects and handled as such; they were normally translated *de verbo ad verbum*, in spite of the impossible Latin constructions which would result. No wonder that the translations which came in fact to be used were initially those made from the Arabic.

All the rather diverse evidence appears to call for one plausible interpretation: Through the import of translated learning, Latin Europe procured itself with the fund of learning which it considered its own legitimate heritage, a fund of learning which can be characterized by the term "Christian" in the above-mentioned secular sense. It consisted of such authors which were familiar by name from the encyclopaedia of late Antiquity and the early Middle Ages, together with such works which could be considered commentaries, explanations or continuations of the tradition.

Watt (1973:150) goes a but further. Noticing that

il aurait été difficile, sinon impossible, de tourner le dos d'une façon purement négative aux musulmans, surtout au moment où l'on avait tant à apprendre d'eux, en science comme en philosophie,

he concludes that

il fallait trouver une contrepartie positive. Cette contre-

partie fut constitué par le recours au passé classique, grec et romain, de l'Europe.

More specifically he argues in connection with the spread of Aristotelianism that

les Européens étaient attirés par Aristote non pas simplement à cause de ses qualités propres, mais également parce qu'il s'apparentait, par certains côtés, à leurs traditions spécifiquement Européennes. En d'autres termes, le fait d'assigner à Aristote une place privilégié dans les domaines scientifique et philosophique peut être interprété comme une des façons qu'avait la chrétienté d'affirmer son autonomie par rapport à l'Islam.

As I shall argue below, this interpretation of the (conscious or subconscious) motives of the Aristotelians of the 12th–13th centuries has a good deal to say about Jordanus and *his* motives.

The wider context of mathematics

Before advancing directly towards this discussion, however, we shall need to return once more to the situation of mathematics as encountered by Jordanus.

The 12th century had not only produced the translations needed to round off the quadrivium. Translators like Gherardo of Cremona, Adelard of Bath, John of Sevilla, Robert of Chester etc. had translated works representing almost all branches of Islamic mathematics. Moreover, besides being known implicitly through these translations, a global picture of (Islamic) mathematics had also been transmitted explicitly to the West through translations of Islamic philosophy, not least through al-Fârâbî's *De scientiis*, which was known from two translations (one due to Gundisalvo, the other to Gherardo¹⁴) and from Gundisalvo's syncretic *De divisione philosophiae*.

Al-Fârâbî's scheme is a product of the old quadrivial scheme and of those developments which had taken place in later Antiquity and during the earlier phase of Islamic mathematics. It divides the subject into seven sub-disciplines: *Arithmetica*; *scientia geometriae*; *scientia aspectuum* (optico/perspective); *scientia stellarum*, "science of stars"; *scientia musicae*, *scientia ponderosorum*, "science of weights"; *ingeniorum scientia*, "science of ingenuities" (Arabic *ilm al-ḥiyal*, "science of artifices/stratagems/devices").

Arithmetic is divided into two: "Active" or practical arithmetic, dealing with concrete number, and used, e.g., in commercial calculation; and "speculative" or theoretical arithmetic, dealing with abstract number - the Old Pythagorean arithmetic as found in Boethius, and in the post-Pythagorean arithmetic of *Elements* VII-IX.

Geometry is also divided into "active" and "speculative". Active geometry deals with the geometry of physical bodies, exemplified in the text by timber, iron objects, walls and fields. Speculative geometry deals with abstract lines and surfaces; like speculative arithmetic it is said to lead to the summit of science. It investigates lines, surfaces and bodies, their quantity, equality or being-greater, position, order, points, angles, proportionality and disproportionality, "givens" and "not-givens" (cf. Euclid's *Data*), commensurables, incommensurables, rationals, surds, and the various species of the latter (cf. the classification of irrationals in *Elements* X).

The treatment of geometry is rounded off by the remark that arithmetic and geometry contain elements and roots together with other matters derived from these roots, and that the roots are dealt with in the *Book of Elements* by "Euclid the Pythagorean".

Perspective "deals with the same things as geometry", but in lesser generality. The details are irrelevant to Jordanus.

The *science of stars* also consists of "two sciences": One dealing with the signification of the stars for future events, and one which is "the mathematical science of stars". In other words, predictive astrology is not counted among the mathematical science (and in fact, it is also said, only counted as a means for judgment and not as a science). The mathematical theory includes, apart from genuine mathematical astronomy, Ptolemean mathematical geography.

Music is, like perspective, irrelevant to Jordanus.

The *science of weights* is treated in only seven lines. One, basic part is the theory of weighing and statics. The other considers the movement of weights, and the means to move them; it investigates the foundations of the instruments by which heavy things are lifted up and on which they are displaced.

The *science of ingenuities* is the preparatory science for the application of the preceding disciplines "to natural bodies, their invention and their rest and their working". So, the term *ingenio* can be read in its double Latin sense, as "cleverness" and as "instrument" (in agreement also with the Arabic text) - we might speak of the discipline as "engineering" or as "applied theoretical mathematics". One subdiscipline *ingenia* of numbers; of these

there are several, including *algebra et almuchabala*, although "this science is common for number and geometry". Under the heading of algebra comes the whole theory of surds, "both those roots which Euclid gives in Book X of the *Elements* and that which is not dealt with there". Geometrical *ingenia* are several too, the most important being construction of masonry. To this genus belongs even mensuration of bodies and "the art of elevating instruments, of musical instruments, and the instruments of still other practical arts, like bows and sorts of arms". Equally "the optical *ingenium*, the art which directs our view to comprehend the truth of things seen at a distance, and the art of mirrors", further the science of burning mirrors; the *ingenium* of the art of very great weights; etc. [not specified]. In the end the contents of the discipline is summed up as *the principles of the civilian practical arts*, like those of masons and carpenters.

Most of the translated mathematical works can be classified under these headings. Often, of course, specific works belong to subdivisions which go unmentioned. So *algorisms*, treatises explaining computation with Hindu numerals (in certain cases expanded to include some commercial arithmetic or algebra); so also spherical geometry (including the theory of the astrolabe) and trigonometry, both belonging with mathematical astronomy.

Even though the substance of al-Fârâbî's arithmetic, geometry, astronomy, and music is to be understood in much greater depth than the corresponding Latin disciplines, they were familiar as members of the traditional quadrivial family. Other parts of the import were known, from Aristotle or from attributions to Ancient authors, to belong to the Ancient heritage (perspective and its subdivisions; science of weights; and spherical geometry).

Some of the imports, however, had to be recognized as genuine non-"Christian" contributions: Algorism was, by the very words of the basic treatise introducing the subject, to be recognized as calculation with Indian numerals; in other treatises which kept silent about the Indians a foreign symbolism could not be covered up in spite of all philosophical commentaries and technical terms borrowed from the schools and the abacus-tradition. And *algebra et almuchabala* was equally foreign, by name and by contents.

Jordanus and illegitimacy. The Jordanian corpus

In general, the mathematical writings of the 12th and 13th

century betray no worry about the existence of non-“Christian” mathematical disciplines. Neither is there any explicit expression of dismal feelings in Jordanus. Still, the total body of his works displays a structure which strongly suggests that he was not only aware of the state of things but also determined to do something about it.

This is a bold statement: Firstly because the canon of Jordanian works is not quite well-established, – but secondly also because several of the definitely Jordanian treatises exist in two or more versions.

To overcome the first problem, I shall build my argumentation solely on such works which can be ascribed with certainty to Jordanus (for which I follow the list established by Ron B. Thomson (1976)). Problems arising from the existence of different versions I shall take up case for case. In order to locate Jordanus’ work with respect to an advanced contemporary understanding of the composition of the science of mathematics I shall classify the writings under al-Fârâbî’s headings.

Arithmetic

As al-Fârâbî, we can distinguish practical and theoretical arithmetic. Concerning the first of these subdivisions, one cannot maintain that Jordanus wrote really practical arithmetic. But he wrote some treatises on a subject belonging to the domain, *viz.*, on algorism. There exist four treatises ascribed to Jordanus and dealing with algorism, two on integers and two on fractions¹⁵.

There can be no reasonable doubt that Jordanus wrote at least one treatise from each set (I shall argue below that he wrote all of them). There are important differences of style between the two sets, but they have one thing in common: Their aim is not to *teach the art of algorism*, as do other algorism-treatises. Instead, they are to *demonstrate from mathematical principles that what is currently done in the art is in fact correct*. In the *Biblionomia* (N° 45), Richard de Fournival speaks of “Jordanus de Nemore’s *apodixis* on the practice which is called algorism”. In the following, where we are repeatedly to meet the same *genre*, I shall borrow Jordanus’ own term and speak of *demonstrationes*.

Parts of the *De numeris datis* (see below) can also be regarded as *demonstrationes* of standard methods and problems belonging to commercial and “recreational” arithmetic (e.g. the rule of three and the “purchase of a horse”).

Under the heading of theoretical arithmetic belongs one of

Jordanus' major works, the *Arithmetica*, presented in the printed edition from 1514 as *Arithmetica demonstrata* (title page) and *Elementa arithmetica* (fol. 1^r). Both titles are justified. The whole domain of theoretical arithmetic, Euclidean as well as Boethian, is collected and ordered in this work; the exposition is founded upon definitions, *dignitates* (a literal translation of the Greek *axíōma* which refers back to the Aristotelian and not to the Euclidean tradition) and *petitiones* (postulates) and organized deductively in propositions.

Each proposition is provided with a demonstration. For this purpose Jordanus uses letters to represent unspecified numbers, instead of the exemplifying numbers used by Boethius and the representation of unidentified numbers through lines in Euclid, al-Khwārizmī and Leonardo Fibonacci. The flavour of the work will appear from proposition I.15:

If a number is divided into two: The product of the whole with itself is as much as the product of each part with itself and of twice one part with the other.

Let the number ab ¹⁶ be divided into two a and b . I say that the product of ab with itself is equal to the product of a with itself and of b with itself, and of twice a with b . For the product of ab with itself is the same as its product with a and b by proposition 113. And the product of ab with a and b is, by the preceding proposition taken twice, the same as the product of a with itself and with b , and that of b with itself and with a . But the product of a with b and of b with a , conversely, is by proposition 8 the same as that of twice a with b . Which is then the product of ab with itself. That of a with itself and of b with itself, and of twice a with b , as proposed.

So, the *Arithmetica*, is a *demonstratio*, as claimed in the printed edition. But it is more than that: The ordering of the material in the 10 books, the start by definitions etc., the coherence and the progressive construction of the presentation - everything shows that Jordanus has tried to write the *Elements* of arithmetic, and hence to elevate his subject, which in the demonstrationless Boethian version had formed the apex of the Latin quadrivium, to the level of Euclidean order (hence providing a basis for a "legitimization" of algebra, cf. below).

Geometry

One major work belongs to this field: The *Liber philotegni*, the *philotegnus* of which is probably to identify either author or user as a "friend of the art" (as *philosophus* is "friend of wisdom").

Before the genuinely geometrical contents of the work comes a metatheoretical introduction, defining (in a way related to the Aristotelian and not to the mathematical tradition) "continuity" (=manifold); "single", "double" and "triple" continuity, "right" and "curved" continuity etc. The definitions are not very coherent, and they are not used in the main text, which instead draws heavily and extensively on Euclid and on a number of other works in the Archimedean tradition.

The work is concerned with advanced and occasionally somewhat specious problems, organized in a loose deductive scheme in series of propositions concerned with similar problems.

While most of Jordanus' treatises contain no clues as to their institutional origin, a few clues are provided here, pointing to the environment of an arts faculty. Firstly, of course, there is the "friend of the art" of the title - but in itself, this could hint simply to the "art of geometry". More decisively are the preliminary definitions. They have no bearing on the mathematical substance of the treatise, and their style is opposed to Jordanus' normal habits (and hence presumably to his personal inclinations). So, they appear to demonstrate that he has felt the need to clear away in advance some metamathematical questions on the status of those plane figures with which he is concerned in the following. The foundations on which he builds the explanations are not related to the Euclidean commentary tradition (nor, for that matter, to the old Latin tradition), but instead to that Aristotelianism which was gradually rising to hegemony at the arts faculties during the first half of the 13th century.

Another hint is the repeated occurrence in the text of a peculiar figure, a *falsigraphus*, who has to be brought to silence. Originally, he descends from Aristotle's *pseudographos*, but he is re- (and mis-)interpreted here (and in other contemporary works) as the opponent who has to be reduced *ad absurdum* - a truly familiar person in the Medieval university disputation.

A final piece of evidence is inherent in a *Liber de triangulis Jordani*. This has long been regarded as another name for a long version of the *Liber philotegni*, but Marshall Clagett's critical edition of both works (1984) showed them to be different though related. Furthermore, his analysis demonstrates that the *Liber*

de triangulis was not written by Jordanus.

At closer stylistic analysis of the work, a wealth of evidence turns up that it was not written on the basis of a manuscript of the *Liber philotegni* as known to us. Instead, it contains (apart from its last part) a student's report of a series of lectures (a *reportatio*) held over an earlier version of the *Liber philotegni*. The lecturer *need* not have been Jordanus himself - but *if* the two persons differ, the lecturer must have had access to Jordanus' own work before it reached its final form as we know it. He must, so it seems, have been a member of that "Jordanian circle" to which I shall return below.

The oral style of the work has conserved some polemical remarks which have no counterpart in Jordanus' written works. The plausibility that the lecturer was either Jordanus himself or some student of his suggests that the polemics reflect Jordanus' own attitudes - and in fact they correspond well to those attitudes which are expressed indirectly in Jordanus' writings. Noteworthy are two references to the assistance of the Almighty; as they stand they can hardly be meant as anything but parody on the Muslim habit to invoke God not only in the beginning and the end of mathematical treatises but often even in the middle of the text. Taking over the Islamic level of mathematical rigour and proof, Jordanus and his circle stay culturally aloof.

Perspective

Nothing but a spurious ascription connects Jordanus' name to this discipline. The last part of the *Liber de triangulis Jordani* (which is not part of the *reportatio* but consists perhaps of notes collected by the lecturer but not covered in the lectures) shows, however, that ibn al-Haytham's extensive work was known in the Jordanian environment.

Science of stars

Nothing in Jordanus' writings reminds even slightly of a reference to astrology.

Even in astronomy, he is not interested in the movement of the stars. Still, he has contributed to the discipline - but once more his contribution turns out to be a *demonstratio*. The work in question is *De plana sphaera*, a treatise on the design of the astrolabe, the favourite astronomical instrument of the time.

The relation of the treatise to the practice of the art is,

mutatis mutandis, pinpointed by Joan Robinson's description (1964:25) of the relation between economists and businessmen: "It is the business of the economists, not to tell us what to do, but to show why what we are doing anyway is in accord with proper principles". Since Antiquity, indeed, the stereographic projection had been used in the astrolabe, and Ptolemy had stated that the equator, the tropics and the ecliptic all projected as circles - but he had given no proof of this, nor had any proof reached the West. Jordanus' treatise teaches nothing different - but he shows Ptolemy's assertion to be truth "in accord with proper principles".

Furthermore, where Ptolemy's *Planispaerium*, the standard treatise on the astrolabe, deals with the location of particular celestial circles, Jordanus generalizes, never speaking of the celestial sphere, equator, tropics, meridians, or ecliptic. The problems of the astrolabe are used as the occasion (not to say the pretext) to construct a general theory of the stereographic projection.

This is a move back toward the style of Ancient spherics. This discipline, too, had been a pure mathematical science, the applicability of which it was in principle left to the reader to discover. Islamic workers in the field had normally been dissatisfied with this opaque purity and had made their own versions of the works, referring constantly to the material implications of the theorems. In view of his broad knowledge, Jordanus must be assumed to have been familiar with these Islamic departures from Ancient style and ideals, and his own style can then be seen as a conscious reaction or correction to this deviation from Ancient norms.

The *De plana sphaera* exists in three versions. In versions II and III, foreign hands have inserted explanations of the "astrolabic" implications of the theorems. So, the way back to Ancient ideals accomplished by Jordanus was traversed immediately in opposite direction by his adaptors.

Science of weights

This is the one field where Jordanus' work gave rise to a genuine school. One work (*Elementa super demonstrationem ponderum*) can be ascribed with full certainty to Jordanus, and another (*De ratione ponderis*) with almost full certainty. The purpose of the *Elementa* was summarized so by Joseph E. Brown (1967a, abbreviated from 1967:3f):

... the catalyst for the medieval "auctores de ponderibus" was a short treatise of Hellenistic origin on the steelyard, *De Canonio*. The Greek work appealed to other treatises, lost since antiquity, to justify its assumptions. Jordanus attempted to supply the missing justification in an exceptionable, nine proposition treatise that often precedes the *De Canonio* and comes to be joined to it. Its starting point was an Aristotelian, dynamic principle applied to the balance and its high point a mathematical statement of the principle of work to prove the inverse law of the lever. Jordanus seems to have worked hastily and loosely so that his treatise became a further catalyst to commentators.

The title of the work, including the terms *elements* and *demonstration*, betrays the usual Jordanian concerns and agrees well with Brown's account. It seems as if Jordanus did here exactly the same thing as he did by his *Arithmetica* for algebra (except that in the case of algebra he had to re-create that discipline in order to obtain agreement between the arithmetical "elements" of the field and their advanced, algebraic applications - cf. below).

The *De ratione ponderis* is a more rigorous extension of the *Elementa*.

Science of ingenuities

Jordanus was no engineer. But he certainly wrote works belonging under the heading *ingeniorum scientia* as al-Fârâbî understood it: Not "practical" mathematics but "theoretical mathematics" especially prepared to serve applications if practitioners would want to. So, the *De plana sphaera* could be classified as an astronomical *ingenium*; and in the *Liber philotegni* one finds a series of propositions on circular arcs (growing out from the trigonometry of chords) which comes to serve both in Jordanus' own *Elementa* and in the *Liber de motu* by one Gerard of Brussels, in spite of its apparently pure character.

Both of these cases stand, however, at the boundary between the disciplines where I discussed them, and the *ingeniorum scientia*. I guess 13th century readers of al-Fârâbî would have classified them as I did. One of Jordanus' major works belongs, however, to a discipline which al-Fârâbî locates unambiguously as an *ingenium*: The *De numeris datis*, which is not only a work on algebra but also a work written in hidden dialogue with the Islamic algebraic tradition.

To a first inspection it does not look so. Even the title, recurring in every proposition, places the work in the tradition of the Euclidean *Data*. The propositions are, with a few variations of form, of this sort: If for certain numbers x, y, \dots certain arithmetical combinations $C_1(x, y, \dots)$, $C_2(x, y, \dots)$, ... are equal to given numbers, then the numbers themselves are given. Everything appears to be concerned with generality, solvability and uniqueness, not with solution for particular values of the given numbers. In many cases the discussion is even stopped at the point where the problem has been reduced to one previously dealt with. Only as illustrating *examples* are the generalizing propositions followed by numerical algebraic problems approaching those known from other algebra-treatises from the time.

The same interest in solvability and generality is found occasionally in Abû Kâmil and Leonardo Fibonacci, but only as exceptional deviations from the normal style. *Their* algebraic works, and all others which could be known to Jordanus, concentrate on the solution of particular problems, aiming at conveying thereby an *implicit* understanding of the general method.

In his methods, too, Jordanus follows other paths than those of the Islamic algebrists. Neither their rhetorical exposition nor their fixed algorithms for the solution of standard cases (nor for that matter the geometric methods used to justify these fixed algorithms) can be traced in his text; instead, Jordanus uses the letter symbolism and the propositions of his *Arithmetic*. Obviously, *his* work on given numbers bears the same relation to his arithmetical *Elements* as Euclid's *Data* to the *Elements* of geometry. It is no simple further step in the track of an already established algebra-tradition.

Still, a closer investigation of the text turns much of this upside down. A fair number of problems and problem types, including rather specious problems and the specific values of the given numbers, coincide with well-known problems from the Islamic tradition. The possibility of random coincidence can be safely disregarded. Jordanus appears to have known not only al-Khwârizmi but also Abû Kâmil and probably even Leonardo Fibonacci. His omission of all names (and even of the name of the discipline) is striking¹⁷.

At the same time, the fair number of full agreements should not make us forget the still larger number of propositions which have no antecedent. Jordanus does not merely offer an alternative formulation of a set of cherished problems, or a *demonstratio* of current algebra. As in so many other cases, he builds up a coherent structure of his own, in which his re-formulations of old problems suggest analogies and extensions which had not

been scrutinized before.

Summing up, we may characterize the *De numeris datis* as follows:

Firstly, it is a *demonstratio* consolidating the current usages of a practice - just like the algorisms and the *De plana sphaera*. Whether the traditional methods are considered sufficiently supported by arguments may be a matter of taste and epoch; but there is no doubt that Jordanus' treatment of the subject is *more rigorous*.

At the same time, Jordanus' treatment is also a *transformation* of the subject. The ordering of the material in most precursors is rather confusing when not chaotic. If not comparable to the strict progress of Euclid's *Elements*, the *De numeris datis* supports comparison with his *Data*. So, the *De numeris datis* transforms a mathematically dubious *ingenium* into a genuine piece of mathematical *theory*.

Finally, it is worth noticing which sort of theory is created. As already pointed out, algebra is brought under the sway of theoretical arithmetic, the discipline *par excellence* of the Latin quaoivium - while for al-Fârâbî algebra was "common for number and geometry", as we remember. Algebra, a bastard by cultural as well as by disciplinary standards, had been legitimized (in such a way that only those who explicitly wanted to could trace the illegitimate origin through the numerical illustrations). In addition theoretical arithmetic itself was brought still closer to classical ideals. Jordanus had already given it its *Elements*; now it was also provided with its *Data*. In spite of the new, shattering claims on mathematical rigour imported during the 12th century, *arithmetica* could still claim to be second to none of her sisters.

Algorism revisited

This finishes the list of indubitable Jordanian works. Before discussing the over-all character of the Jordanian *corpus* we shall, however, return for a moment to the algorism-treatises, certain features of which are better discussed in the end of the list than in its beginning.

It will be remembered that two sets of algorism-treatises exist, one short and one long. A contentual analysis shows beyond doubt that the short set is original and the long set a revision.

The curious thing is that it is the revised version which looks Jordanian. Most striking is the construction of the proofs. The structure of the proofs is the same in both sets, - but while the

revised recension makes use of Jordanus' normal letter-symbolism, this technique is lacking (and wanted!) in the first recension, except in cases where, e.g., an unidentified five-digit number is represented as *abgde*. It looks as if the original recension is a juvenile work, already containing in germ the ideals which are expressed in the mature *opus* (cf. below), but not yet in possession of the techniques permitting it to live up to these ideals. When the techniques had been created, Jordanus returned to the subject and rewrote the treatises as he was now able to.

This interpretation is of course only tenable if the short treatises are indeed due to Jordanus' hand; the best argument that they are is that they express *specific* Jordanian ideals (together, of course, with the ascriptions to Jordanus).

This can be seen at different levels. Firstly, the basis for the proofs is, as in other works, Euclidean theoretical arithmetic. Secondly, the treatises are *demonstrationes*, but also more than that. The tendency to substitute for the mere *demonstratio* of a practice a mathematical generalization is already found in the first treatise on fractions. It does not argue mathematically for the usual manipulations of sexagesimal fractions, no more than the *De plana sphaera* argues explicitly about the heavenly sphere. Instead it invents a generalization of the sexagesimal fractions, replacing the powers of 60 as denominators by the powers of any integer (>1). Moreover, this generalization is made non-trivial by a further generalization opposing these "con-similar" fractions to "dissimilar" fractions, where the sequence of denominators is made up by a gradually extended product of different numbers. These are in fact nothing but the *ascending continued fractions* known from works translated from the Arabic ("two fifth and one third of one fifth" instead of "seven fifteenth"). So, the author of the short treatise creates a theoretical framework encompassing not only the legitimate sexagesimal fractions but also the more dubious *partes-de-partibus*-usage, thereby assimilating the latter into the "legitimate" tradition.

That legitimization is indeed aimed at appears clearly from the revised recension. There, a large section of the introduction explains the concept, not by reference to the Islamic treatises where it belongs but through reinterpretations of material from the Latin (Isidorean and computus) traditions, where it had never been in use.

A similar trick is used in the short recension in order to legitimize the whole dubious subject of algorism. The introduction to the short treatise on integers refers to the Ancients and

contains large passages in the Boethian tradition. "Algorismus" himself (i.e. al-Khwârizmî) is bypassed in silence, reappearing only later when he is given credit for a specific subtlety (as if only this trick was of gentile origin - a strategem which was also used in the *De numeris datis*).

In one way, several of the algorism-treatises depart from normal Jordanian stylistic ideals, viz. by possessing long, discursive introductions. This feature, as the lack of letter-symbols in the short recension, suggests an early origin for the treatises. It would be no wonder if a young teacher in the beginning of his career were still impregnated with the stylistic ideals of this environment - and these were, like the introductions to the short treatises, discursive and heavy with Boethian phraseology.

As in the case of the *De plana sphaera*, Jordanus' followers appear to have missed his specific points. One "Master Gernardus" wrote an "Algorismus demonstratus" which used Jordanus' technique of proofs, and which became quite popular. Significantly, however, he reduced the scope of the work to that of a *demonstratio*: He dropped the dissimilar fractions, and instead of consimilar fractions in general he dealt with the specific sexagesimal fractions.

Jordanus' achievement

We have now finished our tour through the Jordanian *corpus*. The stylistic landscape was not quite homogeneous: Discursive and Boethian in the early *algorism*, slightly shaped by the philosophical context in the *Liber philotegni*, terse in most other works. Still, a coherent picture appears to emerge, a picture which would not change even if a few dubious items had been included in the touring plan.

The important general features of the picture are these:

1. Jordanus wrote *demonstrationes*: Works which, in Joan Robinson's words, give mathematical reasons why "what we are doing anyway is in accord with proper principles". Normally, however, a Jordanian *demonstratio* is generalizing and abstract, hinting little if at all at the specific applications - cf. (3).
2. Jordanus wrote *elements*, works constituting a coherent mathematical basis for a theoretical subject.
3. Jordanus made pure mathematics - or, better, *theoretical* mathematics, in the Ancient sense repeated by al-Fârâbî: mathematics considering abstract principles, not concrete applications or sensible entities.

4. Jordanus worked on subjects belonging under most of al-Fârâbi's subdivisions, i.e. over the whole width of mathematics as understood in his days. But everywhere he would seek out what called for *demonstratio* and *elements*, and regardless of the starting point and the occasion the result would be theoretical mathematics.

5. Jordanus followed the mutual ranking of the mathematical disciplines current in the Latin quadrivium, putting *arithmetic* at the top of the scale. Methodologically, he pursued the standard of Euclidean rigour and organization which had been set by the translations of the 12th century.

6. Combining features (1) to (5), Jordanus succeeded in subsuming the main mathematical currents at hand under the ideal conception of the Latin mathematical tradition, "legitimizing" thereby all dubious subjects.

7. On the other hand, subsuming all important currents and raising of arithmetic, that traditional *pièce de résistance* of the Latin quadrivium, to the best methodological standards, he made the structure of legitimate mathematics competitive on any intellectual market, in agreement with the motto "anything you can do, I can do better".

Motto apart, there is little doubt that Jordanus was conscious of most of this. He need not have *pursued* competitiveness, - he may just as well have made mathematics as he thought mathematics should be. But in a scholarly environment more impregnated with metatheoretical discussions than with genuine mathematical innovation he must surely have known that he was pursuing a particular standard.

So, Watt's explanation of the attractiveness of Aristotle seems certainly relevant as an analogue when Jordanus' work is concerned, especially regarding feature (7).

Feature (6), on the other hand, reminds of another aspect of the reception of Aristotle: The Papal order of 1231 to prepare an inoffensive edition of Aristotle's books on nature (cf. above):

... since, as we have learned, the books on nature which were prohibited at Paris ... are said to contain both useful and useless matter, lest the useful be vitiated by the useless, we command ... that, examining the same books as it is convenient subtly and prudently, you entirely exclude what you shall find there erroneous or likely to give scandal or offense to readers, so that, what are suspect being removed, the rest may be studied without delay and without offense. (Translation Thorndike 1944:40)

The Papal committee never set its pen to paper. The legitimation of Aristotle was only carried out by Albert the Great and Saint Thomas with a delay of some twenty years; anyhow, *their* achievement put a decisive mark on Latin Europe for centuries.

In mathematics, no books had ever been forbidden. Algebra and algorism had never given rise to learned heresies as had the reading of Aristotle. Still, *mutatis mutandis*, Jordanus produced the mathematical analogue of the Albertian paraphrases and the Thomist synthesis. As theirs, his work is an exponent of the great cultural conflicts of the 13th century.

Failure and its reasons

And yet, in spite of this apparent harmony with the dominant mood of his times, Jordanus' influence was not only small compared with that of the Great Dominicans. It was virtually absent.

Truly, a "Jordanian circle" existed in Paris at some period during the first half of the 13th century, which must in all probability have formed around the Master himself, and with which several well-known scholars have been in contact ¹⁸: Richard de Fournival, Campanus of Novara, and Roger Bacon; to these come Gerard of Brussels, author of *De motu*; presumably that "Master Gernardus" who wrote the *Algorismus demonstratus*; the student who wrote down the *Liber de triangulis Jordani*; and perhaps even the authors of some commentaries to Jordanus' statistical works and the adaptors of *De plana sphaera* II and III. Furthermore, Jordanus' works gave rise to a minor current which was never quite forgotten; The *Algorismus demonstratus* may have gained a certain popularity; and throughout the Middle Ages, the *science of weights* was marked by its Jordanian beginnings. But still, the Jordanian current remained insignificant: Even in Paris, the original domicile of the Jordanian circle and the place where Jordanus himself must in all probability have worked, the rumours reaching in the late 13th century the ears of an ordinary student reflect only his works on statics and, probably, the *Liber philotegni*; and during the first 50 years of bookprinting, only one of his works was printed in a single edition (the *Arithmetica*); Boethius' *Arithmetic* was printed thrice, while Albert of Saxony is represented by 13 editions (10 of his *De proportionibus*), and Ptolemy by 12; outside mathematics and related subjects, Albert the Great is represented by 153 editions¹⁹.

Worse: Jordanus' rare followers missed his specific point.

They accepted his theorems, the letter-symbolism for numbers, and his rigour (I disregard Roger Bacon who missed even this part of the message) - but the spread of versions II and III of the *De plana sphaera* and of the *Algorismus demonstratus* shows that they did not share his care for Ancient purity and standards. Richard de Fournival and Campanus may have grasped Jordanus' project - but they demonstrate in their own works and interests that if they understood his ideals they were not inspired by them.

In part, the insignificance of Jordanian mathematics is of course to be explained through a relative lack of interest in mathematics in Medieval learning from the mid-13th century onwards. Even inside mathematics, however, Jordanus was a relatively lonely figure, and at that ill understood by his followers (as we have seen). How could such a situation arise?

If one reflects once more upon the style of Jordanian mathematics, it becomes obvious that Jordanus was lonely for good reasons. Those mathematical works which really gained popularity from the late 12th century onwards were discursive and metatheoretical - precisely in the vein of an introduction to the algorism on integers which Jordanus deleted when revising the treatise. Such is the character of the popular ("Adelard II" and Campanus-) versions of the *Elements*; such is also the character with which the non-Jordanian revisions of the *De plana sphaera* tried to impregnate the work, in order to make it agree with current style and intellectual demands. The evidence can be multiplied *ad libitum*.

Those *Elements* which spread widely did so because they were aimed at and fit for an actual social community, a quasi-profession the demands of which it met: The community of present and former *Masters of arts*. As it appears from his first algorism-introductions, Jordanus had grown out of this community (from where else should he have come?), and he had been impregnated by its ideas and ideals. But growing, he had grown away from it. His mathematics soon stopped being didactical and philosophical; it became *pure mathematics* almost of the Ancient brand - cf. even his references to the Ancients in the early didactical introduction. His standards were defined by those Ancient mathematical works to which he had access; so, they reflected the needs and the structure of a social community since long deceased: The circle of corresponding mathematicians connected to the Alexandria Museum and schools (cf. Høyrup 1980:46). Seen from this angle, Jordanus stands out as a Don Quichote, fighting a fight in which nobody really needed his victory. But even a Don Quichote expresses the ambiguities of his time, may he seem

an anachronism - and in his striving towards anachronistic ideals Jordanus expresses another aspect of Western Medieval culture: The constant longing for a *renaissance* of Ancient splendour and the ever-recurring tendency to produce "renaissances" by every revival at least from the 7th century Visigothic prime onwards: "Carolingian Renaissance", "Ottonian Renaissance", "12th century Renaissance", - and finally the *real* Renaissance. Jordanus' loneliness expresses symptomatically that the cultural glow of the 13th century (and especially that of university learning) was less of a renaissance, i.e., less of a digest of Ancient culture on the conditions of the day, than most Medieval flourishings, and correspondingly more of its own (just the reason why the real Renaissance had to react so sharply against its vestiges), - while on the other hand Jordanus was of real renaissance character.

Epilogue: Epistemology

It is not customary that miners follow the iron-ore to the forge in order to work it up, nor are smiths necessarily the ones who know best how to use the tools they produce. Similarly, I may not be the one who sees most implications of the preceding study for the creation of a general theory of epistemological development, or who can state them with greatest sharpness and clarity. Still, since it was made as an investigation of the evolution and moulding of knowledge as a socio-historical process, I may perhaps be permitted to point out two sets of implications which seem important to me.

The first has to do with the very character of our concept of knowledge. It can be stated in the aphorism that *the way we know is an integrated aspect of our knowledge*. This may seem a rather empty truism, but it is my claim that it is neglect of this truism which has led to the consideration of mathematics as "the hard case" for sociology of scientific knowledge.

To exemplify this, we may consider algebra. If we concentrate exclusively on the "contents"-aspect, we will, e.g., be led to collect all "homomorphous" second-degree problems into a common featureless heap, whether they deal with concrete or abstract number, or with concrete or abstract geometric quantity; we will also be led to identify all treatments of such problems where only the solutions follow algorithms containing the same numerical steps. As long as we accept that the number of dots in the pattern

⋮⋮⋮ is the same as the number in the pattern $\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$ irrespective

of time and culture (I do), this opens no place for sociological understanding of the evolution of knowledge; the process becomes one of *discovery* of discovery-independent truths, and only the cadence and the gross direction of voyages of discovery become problems open to sociological attack.

In Jordanus' case we see that the concentration on the aspect of contents would have made it impossible to discover the most fundamental differences between the *De numeris datis* and the Islamic algebras. It would have closed our eyes to the elevation of the subject to Ancient, "Euclidean" rigour, and to the exclusive subsumption under arithmetic; these, however, were precisely the aspects which had general parallels in scholastic culture, and which hence called for explanations through socio-cultural forces.

This could in itself be said to be of minor importance for a theory of the evolution of mathematical *knowledge*, - unless the quest for coherence and system (itself a "Euclidean" and hence a "formal" feature) had led Jordanus to take up a number of problems which had not been investigated by his precursors. So, contentual innovation is a consequence of the organization, shape and metatheoretical ideals of mathematics, and hence *indirectly* of those forces which shape these; it is the search for *direct* explanations which turns out to be a hard case (and often impossibly hard).

This two-level-approach to knowledge may be insufficient in the case of empirical and technologically testable sciences (and in definitely non-empirical sciences like theology and in the pseudo-sciences - I shall not venture into sharp distinctions in this delicate field). But it can be taken for granted, I assume, that a *one-level* approach to any system of socially organized and epistemologically coherent knowledge is as impossible as in the case of mathematical knowledge.

It can also be observed that the two-level description is an approximation which hardly deserves as much as the (amply misused) name of a "model". Here as elsewhere, an Aristotelian distinction between "form" and "contents" helps us remember to - distinguish; and here as elsewhere we must be aware that we should not stop at a distinction by simple dichotomy, not even dialectical dichotomy.

Above, I spoke of the explanation of epistemological evolution through unspecified "socio-cultural forces". No study can claim to discover and describe all effective forces, nor can a single case be expected to display all generally important types of

“force”. What can be done through a case study is to pinpoint some forces which in the specific case appear to have been fundamental, and to investigate their interplay. This is, I hope, what I have done in my study of Jordanus. The presentation was kept close to phenomena, and therefore a short systematic summary may be useful. My second set of implications is simply that such forces *can* be important, and that matters are *at least* as complicated as this if we want to understand the evolution of scientific knowledge.

Jordanus was part of a general culture, and shared a number of its values. The longing for a “renaissance” and the awareness of an “ethnic” (and maybe religious) “Christian” identity appear to shape his whole project and to pop up in the few scattered non-mathematical remarks of the works. But he expresses the general values in specific form (everybody does!), and *some* wide-spread attitudes and values appear to have been explicitly discarded. So, in contrast to the relique-orientation of his ages (which concerned secular as well as sacred reliques) he did not regard (or at least did not treat) the works of Ancient authors as sacred. *His* renaissance was to be a fresh birth, no resurrection.

Jordanus was also part of a scholarly culture, and must have been brought up in a particular professional environment, - that of incipient universities. Here again, he shared some values and rejected others. His accept of the norms of the “Latin quadrivium” is such an accept of traditional values, but also a rejection of contemporary tendencies to displace the quadrivial arts through Aristotelian philosophy as the summit of secular learning. But also his obvious definition of himself as a “pure” mathematician was a rejection, namely of the current tendency to integrate all Liberal Arts; his replacement of traditional Latin mathematics through works oriented after Ancient standards was an accept of the “new learning” and a rejection of scholarly traditionalism, while his suppression of all traces of illegitimacy goes against the normal habits of that same new learning (although hidden affinities of the same character were revealed in its general pattern). So, whatever he did can in some sense be said to be “wrong” with respect to some forces in his professional environment and “correct” with respect to other forces. The way he oriented his work in this contradictory situation can be assumed to depend in part on his personal history; but it was certainly also dependent on his *access to and understanding of previous knowledge*.

Indeed, Jordanus was a great mathematician. He was able to read from the classics not only their theorems or specific

techniques but also a global view, and to assimilate a thought-style developed in response to a professional social situation which had disappeared in the far past. He was able to read the Ancient works *as mathematics*, with all the implications of an Ancient or modern understanding of that word - prominent among which is the impossibility to read the same works as *authorities* to be accepted on faith.

This fascinating understanding of the possibilities of mathematics will certainly have been an important factor for Jordanus when he oriented himself, consciously and unconsciously, with relation to the contradictory forces and claims of his environment as to the nature of "good" knowledge and thought-style. On the other hand, Jordanus was no Archimedes dropped accidentally in a High Medieval cradle or Arts Faculty. Even when understanding Ancient mathematics *as mathematics* he understood it in part on premisses of his time. It will have been his Medieval upbringing which oriented his thought so firmly on arithmetic that he was able to build up a Euclidean understanding of that field and to develop an adequate technique of proof dealing with general, unspecified number directly and not through geometric mediation.

If we relate this observation to the biological metaphor of evolution we see that past knowledge cannot be given the status of a fixed gene-pool. Knowledge cannot function in the evolutionary process as long as it stays up in Popper's Third World. It will have to *be known* by somebody, and nobody - be he as oriented toward ancient models as Jordanus - is able to know in complete abstraction from the conditions of his own upbringing, where values, connotations and colour were given to his conceptual world. The process can be said to possess neo-Lamarckian or teleological features. The requirements of the time are co-determinants of the interpretation and organization imposed upon transmitted knowledge (when this knowledge attains its effective form in the heads of humans).

Also in another connection is it important to remember that knowledge is only effective when known by somebody. Much of the knowledge developed by Jordanus turned out to be ineffective because nobody was able or willing to know it. Because of specific historical circumstances and personal aptitudes, Jordanus was able to develop a form of mathematical knowledge which was largely out of key with the predominant tune of the "tribe" of Medieval university scholars. This can, if one wants to, be described as a "random" (if definitely not "blind") variation with respect to the predominant conditions of the knowledge-carrying and -transmitting environment, - as "random" at

least as the reading of a die according to classical mechanics, which also results from a multitude of uncontrolled influences. The rejection of Jordanian mathematics by the same tribe is, on the other hand, a paradigmatic example of "selective (non-) retention". No matter how "good" was the mathematics created by Jordanus when measured by a presumed abstract scale it remained out of key with the (sociologically well-rooted) norms, ideals and needs of contemporary scholars, who therefore did not care to invest the time and energy required to carry on with the singing. While the statistics of small numbers had created the possibility that a single scholar could deviate strongly from the ways and needs of his fellow tribesmen in his reaction to the requirements and possibilities of the time, the statistics of large numbers assured that the effects of this deviation remained insignificant.

True enough, a small clan inside the tribe was convinced of the qualities of the Jordanian tune and went on singing it. To a restricted extent, Jordanus' manuscripts were continuously copied throughout the Middle Ages because his mathematics was known *in the abstract* to be good. We have also observed the existence of a small circle of followers and admirers. But even they could not hold the key but switched to that of the larger tribe: Campanus took over some of Jordanus' axioms but produced an eminent *Medieval* Euclid; Richard de Fournival had all his works copied but was in reality more interested in medicine, astrology and alchemy; the *Algorismus demonstratus* took over his apodictic technique but displaced the general theory of consimilar and dissimilar fractions through an explanation of the sexagesimal fraction system; and the adaptors of the *De plana sphaera* pushed Jordanus' chastity aside and spoke shamelessly of celestial circles and stars. All this, of course, are not instances of "selective non-retention"; instead it parallels on a larger scale and in more radical form the teleological (mis-)understanding of transmitted knowledge already discussed above on occasion of Jordanus' "arithmetization" of the ideals of Ancient mathematics.

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NOTES

1. This question is the subject of Booß & Høyrup 1979, which investigates a series of concrete exemplifications from the physical as well as the social sciences.
2. Høyrup 1980:9. A more full presentation, connecting the discussion to the problem of "quasi-Aristotelian" vs. Humean causality and to the "externalism"- "internalism"-debate is Høyrup 1982 (in Danish).
3. An excellent and recommendable exposition of this theme is Werner 1976.
4. Høyrup 1988.
5. In order to make my contribution to the present volume less bulky I shall take advantage of the existence of this complete study, omitting most of the detailed justifications for specific conclusions and many bibliographic references here. Detailed discussions, justifications and extensive references will be found in the complete version.
7. All these decrees will be found in Denifle and Chatelain 1889.
8. The prohibition is found in Denifle 1885:222, cf. pp. 188-191.
9. *Physica* I, I, i. In order to guard the proportions of the intellectual change of climate one should keep in mind that not all members of the Dominican order had attained the same favourable attitude to philosophy by 1250 - cf. van Steenberghen 1966:276f.
10. Both passages continue to be quoted in elementary mathematical treatises down to the 13th century.
11. Often, the device goes under the name of "Gerbert abacus". The first references, however, antedate Gerberts' mathematical activity by a few years; most probably it is a result of the first tender Arabo-Christian scholarly contacts in the Lorraine.
12. And yet, Héloïse and Abaelard the greatest of dialecticians baptized their little son Astrolabius.
13. So, in the 12th century Castilian epos *Poema de mio Cid* (I, 31), the hero decides to keep his prisoners as servants "because we cannot sell them, and we shall gain nothing from decapitating them".
14. In the following, I follow Gherardo's translation (with a single instance of recourse to the Arabic text).
15. Original, "short" set: *Opus numerorum* on integers and *Tractatus minutiarum* on fractions. Revised, "long" set: *Demonstratio Jordani de algorismo* and *Demonstratio de minutiiis*.

16. By *ab*, Jordanus means the sum of *a* and *b*. This is his only touch of algebraic formalism. His letters serve the purpose to represent unspecified numbers, and not symbolic abbreviation.
17. The taciturnity amounts to conscious deceit. In fact, in *one* place a method is referred to "the Arabs". This regards an alternative method for a problem of "purchase of a horse" (borrowed perhaps from Leonardo, and stripped of its concrete dressing) - as if not the whole subject had been borrowed from "the Arabs"!
18. The arguments for this are utterly complex; they involve among other things access to the same manuscripts, hand-writings and other clues to the origin of manuscripts, and the quality of the information to which various authors had access. Since it is hopeless to make a brief but still concrete sketch of the argumentation, I shall only refer to the complete version of the investigation.
19. Truly, many of the Albertian ascriptions are spurious; still, if we are out for an estimate of the fame of an author this plays no role (we might even claim that spurious ascriptions ought to count doubly, fame having in this case to compensate for lack of true evidence).

BIBLIOGRAPHY

It should be observed that the following bibliography contains only that part of the secondary literature which is explicitly cited in the above condensed version of the study. Some 85% of the items from the full bibliography have been deleted, among which many which have been of fundamental value for me; for reasons of space I can only refer to the bibliography in the complete version.

Similarly, the list of sources contains only Jordanus' own works and such other sources which are quoted directly above. Once again, I shall refer to the complete version for fuller information.

Sources

- Albert the Great, *Physica*. In: *Beati Alberti Magni ... Operum tomus secundus*. Lyon, 1651.
- Anonymous, *Liber de triangulis Jordani*. Critical edition in: Clagett 1984.
- Anonymous, *Poema de mio Cid*. Sexta edición, Madrid: Aguilar,

1963.

- al-Fârâbî, *De scientiis*. Arabic text with Spanish translation, together with Gherardo's and Gundisalvo's Latin versions in: al-Fârâbî, *Catálogo de las ciencias*, ed. A.G. Palencia. Segunda edición, Madrid, 1953.
- Isidor of Sevilla, *Etymologies*. In: *Patrologiae cursus completus, series latina*, accurate J.P. Migne, vol. 82.
- Jordanus, *Arithmetica*. In: *In hoc opere contenta: Arithmetica decem libris demonstrata ...* ed. Jacques Lefevre d'Étaples, Paris, 1514.
- Jordanus, *De numeris datis*. In: Jordanus Nemorarius, *De numeris datis*. A Critical Edition and Translation by Barnabas B. Hughes. Berkeley: University of California Press, 1981.
- Jordanus, *De plana sphaera*. In: *Jordanus de Nemore and the Mathematics of Astrolabes: De plana spera*. An Edition with Introduction, Translation and Commentary by Ron B. Thomson. Toronto: Pontifical Institute of Mediaeval Studies, 1978.
- Jordanus, *De ratione ponderis*. Critical edition in: Moody & Clagett 1952.
- Jordanus, *Demonstratio de algorismo*. Description and excerpts in: Eneström 1906.
- Jordanus, *Demonstratio de minutiis*. Description and excerpts in: Eneström 1907.
- Jordanus, *Elementa super demonstrationem ponderum*, Critical edition in: Moody & Clagett 1952.
- Jordanus, *Liber philotegni*. Critical edition in: Clagett 1984.
- Jordanus, *Tractatus de minutiis*. Description and excerpts in: Eneström 1913.
- Richard de Fournival, *Biblionomia*. Published in: L. Delisle, *Le cabinet des manuscrits de la Bibliothèque Nationale II*. Paris, 1874.

Modern authors

- Boncompagni, B., 1851, "Della vita e della opere de Gherardo Cremonese, traduttore del secolo duodecimo, e di Gherardo da Sabbionette astronomo del secolo decimoterzo", *Atti dell'Accademia pontificia de' Nuovi Lincei* 4 (1850-51), 387-493.
- Booß, B. & J. Høyrup, 1979, "From Kepler's Planetary Motion to the 'Quark Confinement' in Elementary Particle Physics, Empirical Investigations in the Problem of Model Transfer and Analogy", *Materialien des Universitätsschwerpunkt Mathematisierung der Einzelwissenschaften, Universität Bielefeld* 14, 211-242.
- Brown, J.E., 1967, "The *Scientia de ponderibus* in the Later

- Middle Ages", *Dissertation*, The University of Wisconsin.
- Brown, J.E., 1967a, [Abstract of Brown 1967], *Dissertation Abstracts International A* 28 (1967-68), 3097.
- Cantor, M., 1900, *Vorlesungen über Geschichte der Mathematik*, II. 2. Auflage, Leipzig: Teubner.
- Clagett, M., 1984, *Archimedes in the Middle Ages*, Vol. V, *Quasi-Archimedean Geometry in the Thirteenth Century* (Memoirs of the American Philosophical Society, 157 A+B), Philadelphia: The American Philosophical Society, 1984.
- Denifle, H., 1885, "Die Constitutionen des Prediger-Ordens vom Jahre 1228", *Archiv für Literatur- und Kirchengeschichte des Mittelalters* 1, 165-227.
- Denifle, H., & E. Chatelain (eds.), 1889, *Chartularium Universitatis Parisiensis*, I, *Ab anno MCC and annum MCCLXXXVI*, Paris, Frères Delalain.
- Eneström, G., 1906, "Über die 'Demonstratio Jordani de algorismo'", *Bibliotheca Mathematica*, 3. Folge 7 (1906-1907), 24-37.
- Eneström, G., 1907, "Über eine dem Nemorarius zugeschriebene kurze Algorismusschrift", *Bibliotheca Mathematica*, 3. Folge 8 (1907-1908), 135-153.
- Eneström, G., 1913, "Das Bruchrechnen des Nemorarius", *Bibliotheca Mathematica*, 3. Folge 14 (1913-1914), 41-54.
- Günther, S., 1887, *Geschichte des mathematischen Unterrichts im deutschen Mittelalter*, (Monumenta Germaniae Paedagogica III), Berlin.
- Høyrup, J., 1980, "Influences of Institutionalized Mathematics Teaching on the Development and Organization of Mathematical Thought in the Pre-Modern Period", *Materialien und Studien, Institut für Didaktik der Mathematik der Universität Bielefeld* 20, 7-137.
- Høyrup, J., 1982, "'Videnskabens antropologi' - et essay om 'ydre' og 'indre' årsager og deres sammenhæng", in H. Kragh (ed.), *Bidrag til videnskabshistoriens teori*, pp. 29-46, København: Akademisk Forlag.
- Høyrup, J., 1988, "Jordanus de Nemore, 13th Century mathematical Innovator: An Essay on Intellectual Context, Achievement, and Failure", *Archive for History of Exact Sciences* 38, 307-363.
- Moody, E.A., & M. Clagett (eds, transls.), 1952, *The Medieval Science of Weights (Scientia de ponderibus)*, *Treatises Ascribed to Euclid, Archimedes, Thabit ibn Qurra, Jordanus de Nemore and Blasius of Parma*, Madison, Wisconsin: University of Wisconsin Press.
- Robinson, J., 1964, *Economic Philosophy*, Harmondsworth, Middlesex: Penguin, 1st. ed. 1962.

- Thomson, R.B., 1976, "Jordanus de Nemore: Opera", *Mediaeval Studies* 38, 97-144.
- Thorndike, L., 1944, *University Records and Life in the Middle Ages*, New York: Columbia University Press.
- van Steenberghen, F., 1966, *La philosophie au XVIIIe siècle*, Louvain: Publications Universitaires/Paris: Béatrice-Nauwelaerts.
- Watt, W. Montgomery, 1973, "L'influence de l'Islam sur l'Europe médiévale", *Revue des études islamiques* 41, 129-156.
- Werner, E., 1976, "Stadtluft macht frei: Frühscholastik und bürgerliche Emancipation in der ersten Hälfte des 12. Jahrhunderts", *Sitzungsberichte der sächsischen Akademie der Wissenschaften, Philologisch-historische Klasse* 115:5.