CREATIVE RATIONALITY: TOWARDS AN ABDUCTIVE MODEL OF SCIENTIFIC CHANGE

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ABSTRACT

I argue for an abductive model of inference that embraces personal, cognitive processes and the social processes in which new conceptual schemes are negotiated and established, by showing that an abductive schema can be applied to patterns of inference ranging from perceptual inferences involved in the creation of new interpretative concepts to the construction of inclusive conceptual schemes. The advantages of the abductive scheme is that it shows where a novel insight or interpretation fits into a larger framework of mental, physical and social activity. It allows us to provide a rationale that links discovery to inductive and deductive patterns of inference without restricting our accounts to a particular logical form.

1. Is there a logic of scientific discovery?

Discovery implies the disclosure or the introduction of something new: a new molecular structure, say, or a new planet. It can also imply a substantial change in point of view: a gestalt-switch or a paradigm shift. To say that a process is scientific usually connotes some systematic if not logical procedure. But logics are incompatible with innovation. Years of research in artificial intelligence seem to confirm the paradox that Meno set Socrates in Plato's *Meno*. Plato defends Socrates' thesis that all knowledge is recollection by means of a thought experiment in which Socrates responds to a dilemma about inquiry. Someone posing a question either knows the answer (so would have no need to ask) or has no knowledge of the answer (so would not be able to recognize the answer). The questioning is therefore pointless. Meno's uneducated slave boy is asked to construct a square double the size of a given square. In response to further questioning, he can do so. Socrates concludes that his success proves that the boy remembers mathematical knowledge that he had not realized that he possessed. An alternative interpretation is that in the course of eliciting an answer from the boy, Socrates' questioning educated him, enabling him to solve the problem. Many artificial intelligence programs can discover only in the Platonic sense; discovery is reduced to algorithmic procedures which few philosophers and historians regard as genuine discovery.¹ I am interested in the other, naturalistic sense of discovery. Despite optimism about artificially intelligent discovery systems the question — Is there a logic of scientific discovery? — has usually been answered negatively. It is thought that the processes of discovery are too opaque or subjective to investigate. In *The Logic of Scientific Discovery* Popper relegated them to psychology and his translators appropriated the label 'discovery' for purely logical procedures such as justification or testing.

Can we characterise discovery as both rational and as capable of genuine innovation? I propose a positive answer based on the idea of *abductive inference*. I shall attempt to characterise this in a way that brings out a common feature of a range of scientific innovations. These range from thinking about new phenomena that involves manipulating images to arguments about large-scale field-observations. The range of the examples is important because it spans the personal domain of perceptual inference leading to the creation of a new experience and the social domain in which scientists argue for interpretations of evidence that bring about major changes of viewpoint. If both the creative insights of individuals and the public, argumentative strategies of scientists can be characterised in terms of an abductive scheme this may help us to understand what they have in common.

1.1 Interpretations, Pictures and Paradigms

How can we represent discovery as a creative form of inference? The logical models of post-empiricist philosophy and of traditional artificial intelligence have been shown to be inadequate. What is to replace them in the new practice-based, naturalistic philosophy of science?

Kuhn's answer, following thinkers such as Piaget and Wittgenstein, drew an analogy between conceptual frameworks and visual gestalts. Like Hanson, Kuhn asserted the equivalence of perceptual experience, inference and interpretations, likening changes of world-view to switches between ways of perceiving. Paradigms assimilate a great many facts and ideas in the same way that we assimilate features in our visual field by interpreting them in terms of some concept. Anomalies are facts or other features that cannot be assimilated into the overall picture. Scientific change occurs when anomalies accumulate and a new picture is needed. Kuhn and Hanson applied the idea of the gestalt switch to change of any kind — whether the reinterpretation of a discrete set of facts or a wholesale change of worldview. This equates the unconscious formation of a visual gestalt with apparently quite different processes such as the interpretation of a complex array of data or the attempt to theorise a number of mutually incompatible theoretical propositions. My objections to this use of the analogy are that it gives pre-eminence to the visual aspect of perception at the expense of other modes and that it conceals the process behind the change.

Let us examine the position more closely. Hanson and Kuhn argued that a scientific theory presents a view of the world, so, to understand a theory is analogous to being able to see what a picture depicts. On Kuhn's analysis, scientific change consists of alternating periods of normal science (which are rational in the traditional sense favoured by philosophers of science) and occasional bouts of revolutionary, creative change. Philosophies of science tend to emphasise normalcy, the procedural and rational features of science. It has been said that normal science is as close as real life ever comes to the philosopher's notion of what it is to be rational.² Creativity and rationality have been hard to reconcile because creativity involves bending, breaking or flouting agreed, welldefined procedures that guide and define normal science. Kuhn's picture of scientific change perplexed many philosophers because it posited fundamental discontinuities between the different paradigms or worldviews. Studies of visual, procedural and other non-verbal aspects of scientific change show that there is considerable continuity of practice behind changes in vocabulary (as Kuhn himself recognised). If we attend to the whole range of activities involved in producing new knowledge then scientific change becomes, if not explicable, then at least compatible with a cognitive approach.

Nevertheless, Kuhn's approach said little about the thought processes of scientists. His analysis of the transforming role of thought experiments turns on the important insight that these enable changes of viewpoint by

exposing tensions between conceptual schemes. They do this by exposing anomalies, i.e. inconsistencies in the way that one of the schemes relates to the world. These paradoxes are not simply logical contradictions. It may actually be difficult to express them verbally, within the framework of assumptions of the view that is being criticised. I have tried to capture this with the notion of the impracticability of enacting certain procedures.³ This notion highlights the importance of embodied performance to our understanding of both real and virtual worlds. It was absent both from Kuhn's analysis and that of many of his critics.⁴ This limitation persists today in the way that pictureability and the ability to interpret experience in terms of pictures are treated as equivalent to complex cognitive processes of interpretation, understanding and explanation. It is easy to see why this should be: visual experience happens to be that part of our sensory experience for which we can easily make word-pictures, i.e. it is that part of our sensory experience for which verbal modes of expression are well developed.⁵

1.2 Inference as Abduction

Hanson sometimes calls the step in which a new insight or interpretation emerges an *abduction*. An inspection of Hanson's examples shows that some (such as Kepler's conclusion that the orbit of Mars must be an ellipse) do fit a simple abductive scheme of the sort proposed by C. S. Peirce. Others are wholesale, synthetic intellectual achievements to which he applied the label without providing any analysis (e.g., Newton's grand synthesis for the system of the world). No doubt these could appear as abductions if we reduce them to essential moves which fit the schema. Is this an improvement on the schema-fitting of logical-empiricism or of traditional artificial intelligence?

Is abduction profound as well as universal in scientific thought, as Peirce, Hanson and others believed? Since anomalies are essential stimuli to making an abductive inference, we could co-opt Kuhn here as well. De Mey has defended the unity of interpretation, taking the analogy between gestalt-switch and paradigm-switch further than Kuhn or Hanson. He shows, for example that it is quite plausible to interpret the development of Harvey's discovery of the circulation of the blood in terms of an interactive theory of perception of the sort devised to explain the gestalt switch. However his project, like mine, is to explain how the possibility of a switch from an old view to a new one can arise.⁶ More recently Shelley has defended an abductive theory of visual reasoning for the field of archaeology.⁷ He treats recognition, comparison leading to identification, reconstructive identification and visual analogies as examples of abductive inference. I believe it is possible to extend this sort of analysis in terms of a generalised abductive schema. This can characterise scientific change at all levels from the local and personal (such as the perception of a gestalt, and switches between gestalts) to the highly articulated, inter-personal constructs on which shared, public views of the world depend.

By abduction, I mean the general process whereby a conjecture is made, as set out by Peirce:

1. Observe anomaly A

- 2. Abduce H where H implies A
- 3. Test H by induction

(e.g. produce instances experimentally, where $H' \rightarrow A$, $H \rightarrow A$, etc)

A further step would be to

4. Develop a deductive argument such that H entails A.

For Peirce step 4 was important because it articulates an explanation of **A** in terms of **H**. A simple example is Leverrier's inference to the existence of a previously unknown planet from anomalies in the orbit of Uranus. When observed close to the predicted position, the planet Neptune explained the anomalies. Another example is the perception by Harold Kroto and his colleagues that mass spectographs of carbon typically produce peaks at a number of values, in particular multiples of 60 and 70 times the atomic weight of Carbon. These peaks were anomalies until a theory of the possible stable structures for molecular carbon was developed that could show that molecules containing 60 atoms occur.

What's missing from the scheme and the examples is the *context* in terms of which it is agreed that an observation is problematic and which provides sources for the new insight or hypothesis H and also for showing that H explains A.⁸ Consider the ability that we have to make inferences that explain archaeological artefacts. Shelley appeals to the role of our

knowledge about the processes of making such artefacts.⁹ This contextual or background knowledge is essential to the ability to interpret certain broken rocks as having been shaped by human action rather than natural forces. Such knowledge is often invoked tacitly so that the fact that an abduction depends on it will be brought out only through the reconstruction (stage 4). The importance of such knowledge ties in closely with my explanation of the efficacy of thought experiments. These are not the selfcontained arguments they appear to be. They can persuade only if we can perform them, and the ability to perform them depends on experience that thought experimenters bring into the thought experimental situation.¹⁰

The switch from knowledge to experience is significant. Those concerned to provide formal schemata for inferences identify knowledge with what can be expressed in language. But there are other kinds of knowledge. Experiential knowledge may be expressed in other ways, for example through a competent performance. The need to transcend propositions and their strong association with formalism may explain why images and pictures appealed to philosophers such as Kuhn and Hanson. Is it possible to represent such knowledge as an abductive scheme? We have seen that Kuhn and Hanson assumed that abduction can produce a variety of different things: a novel insight (such as the move that resolves the paradox in a thought experiment),¹¹ a change of view (such as Kepler's recognition that the egg shaped orbit for Mars is actually an ellipse),¹² a new concept or experience,¹³ an integrating explanation or world-view.¹⁴

I believe that abduction offers a better model than the 'memorable picture' theory that Hacking advances to explain the efficacy of thought experiments.¹⁵ Hacking argues that various recent attempts to explain the effectiveness of thought experiments are weak: though each may capture something important, neither argument nor embodiment nor mental-modelling add much to our understanding of how they work. Instead he proposes the idea of narratives that create memorable word-pictures. This invokes the 'picture' theory of understanding espoused by Wittgenstein, Kuhn, Hanson and others. It is clear that logic, language and imagery do work together to change our minds and our views about the world, but reasserting the intuitive appeal of pictures does not help. Instead, I want to use the idea of abduction to discriminate between different parts of an argument and between different stages in its history.

I will offer three examples: a thought experiment, large-scale geolo-

gical surveys of the sea-floor, and an account of reasoning with images in laboratory science.

2. Abduction in Thought Experiments

My first example is one of Galileo's best known critiques of the Aristotelian doctrine of motion applied to falling bodies. According to Aristotle heavy bodies fall more quickly than light ones. Imagine that two stones, L and H, are falling. Grant the common sense belief that H falls more quickly than L. Now imagine that H becomes attached to L. Since light objects fall more slowly, L will retard the fall of H. Yet L + H weigh more than H, so the conjoined stones must fall more quickly than H. In this experiment a sequence of events and interventions can cause two contradictory outcomes. The contradiction exposes a fallacious assumption rather than incompetent performance. The experimenter must choose. The problem is resolved if we abandon the postulate that velocity is proportional to weight, so: H = L = H + L. Of course Galileo's narrative may be reconstructed as a deductive argument, but the crucial move — the proposition that "H = L = H + L" — cannot be *deduced* from what has gone before, nor is it an induction.¹⁶ Assenting to it involves insight, a new way of perceiving the whole sequence of events.

This example illustrates how an insight which leads on to a discovery does not reduce to logical procedures.¹⁷ Abduction is not achieved by some combination of inductive and deductive reasoning. But why should it matter whether an insight is *derivable* or not? Platonists such as Brown clearly think it does matter, because they view non-derivability as evidence for the independent existence of the truths that we intuit.¹⁸ Although most thought experiments can be reconstructed as deductive arguments having suppressed premises, this does not explain how the experiment came to be devised.¹⁹ Abduction offers a better model for a naturalistic account, which should offer some explanation of the insight in terms of prior experience, relevant knowledge or stratagems such as making analogies.

3. Abduction in Imagistic Thinking

I now want to apply the abductive model to thinking that involves visualisation. Understanding how scientists invent and enhance the representational capability both of words and visual images may help us understand the investigative practices that use them. The use of graphical representations in the next two examples points towards three common features:

Reduction and Enhancement

The representational power of images develops in the following way: There is first a *reduction* of complex phenomena to an abstract image (usually a pattern or set of patterns). The image is then enhanced by 'adding' dimensions, first to create a three dimensional structure and then — where a causal explanation is sought — to construct a real-time (4-D) or process model. I call the progression from two- to four dimensions, *dimensional enhancement*. The process is actually more complex than this summary suggests, since the 2-D images with which the process begins are themselves abstractions, dimensionally-reduced representations of complex real-time experience. Dimensional reduction is always necessary when recording real world processes as, say sketches in a notebook. Dimensional enhancement therefore depends on a prior reduction. The 2-D images cannot directly represent process, but then that is not their function.

Formal Consolidation

A further feature is that in all cases the initial enhancement is followed by *formal consolidation* in which the initial image(s) and new ones are derived from the process model. This involves reducing the complex images from four dimensions to two. Dimensional reduction is used in both the construction and the consolidation stages. In the consolidation stage reduction makes the dissemination (say, of predictions or observed results) possible, in the form of printed diagrams or photographs. A search for new effects predicted by the model might typically be observed as 2-D patterns rather than full-blown 4-D processes. This stage resembles a deduction, albeit one accomplished through manipulation of objects that are not propositional or symbolic representations. Words and symbols take precedence during the process of formal consolidation in

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which a 4-D model is presented as a plausible explanation for the initial observations, as represented by a selected sub-set of the initial images.²⁰

These features highlight several different roles for images. They correspond to different stages of the process of constructing a new representation and integrating it into an argument:

(i) they may be instrumental in generating new representations or in extending the use of existing ones.

(ii) they symbolise an integrated model of a process that involves many more variables than the eye or the mind could otherwise readily comprehend. This integrative function presupposes the ability to combine information from different senses.²¹ In these two cases visualization is essential to the construction of interpretative and analytical concepts.

(iii) they enable empirical support for the theory embodied by the model, usually through the dissemination of images in 2-dimensional form. Here the visualization of observations or data assists a verbal argument that may have been developed by non-visual means.

3.1 The Modes of Perception

The visual mode of perception does not work in isolation from other modes of perception or from other persons as sources of experience.²² The power of images consists largely in the fact that they integrate different types of knowledge and experience. The ability to integrate information from various sources is crucial to scientific inference.²³ My second example addresses this directly. It shows how visualisation works in conjunction with sensorimotor awareness (proprioception or kinaesthetic awareness) to produce representations whose cognitive (generative) and social (communicative) functions are inextricably linked. Mental models having such integrative power could not have been developed from passive, visual perception.

Identifying functions such as reduction, dimensional enhancement and consolidation suggests a classification of scientific activity according to whether visualization has an essential, ancillary or negligible role in construction, consolidation or dissemination. My argument is that the first parts of the sequence (abstraction, dimensional enhancement and reduction) can be schematised as a form of abductive inference, followed by the consolidation of these steps into a deductive form of argument.²⁴ This schema can then provide a framework that specifies the roles of inductive and deductive forms of argumentation as well as the roles of images and words, in the construction of scientific arguments.

There are obstacles to theorising the cognitive and the social uses of images.²⁵ Some originate in the different emphases of history of science, cognitive science, and of sociology. Historians tend to emphasise the particular contingencies of a situation or sequence of events whereas psychologists and sociologists tend to look for general features, mechanisms and relationships. Cognitive scientists tend to approach representations as mental sorts of objects: the play of images and ideas is a play of the mind; externalised representations are aids to, or provide constraints on, these mental processes. The individualism of cognitive approaches has made them unpopular with sociologists of science whose concerns lie primarily with the public domain in which new knowledge is disseminated. Yet neither approach focuses on how personal experience and mental processes get articulated into visual or verbal languages.²⁶ The sources and constraints are material, cognitive, personal and social. We don't yet know how to integrate these.

There is an enormous difference between the broad scale and the complexity of the accounts offered by historians and the mental processes invoked by cognitive psychologists to explain precisely defined experimental results. While cognitive scientists consider processes that may occupy a few milliseconds, historians typically describe visualizations that develop over much longer timespans. Instead of the real-time experiences of experimental subjects, historians must work with words and sketches - reduced, two-dimensional abstractions. A more systematic approach to describing the uses of visualisations is needed, so that historical accounts of different episodes can be compared. The method proposed here addresses an inherent limitation of narrative accounting, that narratives impose serial order, a sequencing of events quite unlike the multipleconnections and plasticity of imagination expressed by the images and structures that scientists use to represent complex natural processes. It might then be possible to place visual images in the context of the material manipulations and verbal arguments that confer the scientific meaning that they have in published work.

3.2 In here or out there?

To avoid isolating visual representations either from other modes of

perception or from other perceivers it will be helpful to dispense with H. A. Simon's distinction between internal and external representations. This is largely taken for granted in artificial intelligence research. It presupposes a dualist (mind-world) view which, I believe, a better understanding of the functions of visual imagery will render obsolete.²⁷ Simon identified the importance to reasoning and problem solving of the ability to create external representations of the processes. These function both as records and as guides, for reasoning procedures that are too complex to conduct with internal or mental representations. We can apply this insight without having to reduce reasoning to problem-solving or problem-solving to changes of representation.²⁸ Representations must be 'externalised' if they are to communicate well enough to enable discussion and criticism. 'Externalized' representations can take many forms: verbal accounts, drawings, apparatus, photographs, experimental narratives, databases — all feature in the whole range of knowledge-making processes.

Just as a picture may be worth a thousand words (or data-points), so a few words or symbols may eventually come to express many thousand visual, auditory or tactile experiences. As De Mey puts it, for us to perceive an object at all, "sensory qualities need to be combined and brought to bear upon a single entity."²⁹ Observation continues to be an active process whose primary aim is to create shared experience. A similar point has been made about externalised representations in general by Latour (he calls them inscription devices) and about verbal representation in particular in my work on construals and reconstruction.³⁰

4. From Anomaly to Innovation

My next example is a recent episode in geology, the acceptance of a mechanism for sea-floor spreading in relation to Wegener's theory of continental drift. The claim for cognitive relevance of an example from the past assumes that the protagonists in that account have the same cognitive make-up as we now do. ³¹ Nevertheless, geologists' intellectual objectives change and this affects the sort of representations they try to produce. We should therefore note the importance of explanation to this enterprise. As Rudwick points out, not all geology has been concerned with explanation. In the stratigraphical geography that set the stage for

the Devonian controversy, graphical representations were normally twodimensional maps and three dimensional sections. The object was to produce an agreed sequence of strata which could be identified all over the world. Since causal explanations were not being sought, dynamical or process models were not important (nor were they necessary to everyday field practice).³² Whereas the use of maps and sections was largely descriptive in the early 19th century, in later geology such graphical constructs are expected to generate (or tie in with) causal explanations represented through process models such as block diagrams.

In a recent study of the role of data-visualization in the resolution of the controversy over continental drift, LeGrand shows that certain images acquired a crucial, persuasive role.³³ He likens their cruciality to that of results whose decisive status transforms ordinary experiments into crucial ones. The cruciality of a series of experiments is often expressed graphically because, as LeGrand notes, these can combine diverse sets of data and also assist the selection and comprehension of salient information about a process. In this episode a key exhibit is a particular run of the ocean survey ship *Eltanin*. Selected from a large survey of sea floor magnetization, this image became known as *Eltanin-19*.

LeGrand's analogy between the cruciality of images and that of results is apt. My interest in this episode is the construction of what turned out to be a successful model of sea floor spreading, rather than the cruciality of the visualized data acquired from magnetometer scans. I shall summarise LeGrand's account of the process of visualization with reference to the process of generating new representations and, from them, new explanations:

1. During the 1950s measurements of magnetic field strength were made in the form of magnetometer scans along well-defined paths.

2. Records of these scans were combined or accumulated into anomaly maps. An anomaly map displays the *pattern* of magnetization built up by many hundreds of scans representing many thousands of numerical readings. The visualization of data tables as two-dimensional maps involves a translation from numerical into graphical form. A key feature of these patterns is the striping which indicates regular differences in measured field strength. Viewed magnetically, the seafloor in the region of the eastern pacific rise consists of alternating strips of rock, each of which has a different magnetic field strength.

3. The anomaly maps present 'the data' in a form that invites expla-

nation. However the data do not suggest a particular hypothesis (any more than the contradiction in Galileo's thought experiment implies that weight is irrelevant, see section 2). Mapping of this kind is a widely used technique. It highlights data accessed through instruments and it can be used to incorporate other, possibly relevant phenomena and features, such as centres of volcanic activity or earthquakes.³⁴

4. The anomaly maps were used to construct three-dimensional representations which enabled them to incorporate other kinds of information about the structure of the sea floor — e.g., its chemical composition, thickness, temperature, underlying geology and so on. These static models accumulated several different types of information into a single type of drawing which became the new focus of thought and argument. Thought-experimental narratives accumulate and compress information in the same way.

Several explanations of the anomaly patterns were available. These drew on geological and chemical theories as well as knowledge about periodic reversals in the earth's magnetic field. No particular explanation was preferred. There was evidence to support the theory of continental drift but no plausible mechanism had been proposed for the movement of continents. In the mid 1960s Vine, Matthews and Wilson proposed a theory of ocean floor spreading that incorporated the striping shown by the anomaly maps:

• molten basalt is magnetized as it cools;

• its magnetization will depend on the sense of the earth's field at the time it is extruded and cools;

• this magnetization will subsequently affect the field strength in the region above it, being 'added' to or 'subtracted' from the earth's field. Now, assuming that extrusion continues during periodic reversals of polarity, the magnetic striping of the sea floor *can be seen as* a record of these reversals. Both the process and the assumption of polar reversal were thought improbable at this time.³⁵ This example illustrates the richness and the complexity of the knowledge that goes into seeing the anomaly-maps as evidence for a revolutionary new theory.

5. This hypothesis treated the anomaly patterns as a consequence of a geological process whose details could be worked out, making the extrusion of molten basalt a consequence of a much larger set of processes. This is a *process model* which incorporates the static structural representation. The 3-D model now stands for a state (the current state) of the 4-D process, as does the 2-D anomaly pattern. LeGrand calls the set of images associated with the process model a symbolized theory.

At this point, the magnetometer records bore only a loose relationship to the theory, which attracted little support. In 1965 Vine and Wilson inferred that if the process explanation is correct the striping should be symmetrical. If molten basalt is extruded along a fissure or axis, identified as a ridge, patterns either side of the ridge should show mirroring. This prediction tightened the link between the theory and the anomaly patterns.

6. Once they had derived symmetry as a feature of anomaly maps from the process model, Vine and his colleagues then found seafloor scans that displayed this property. One of these — the 19th run of the survey ship Eltanin — displayed it particularly well. Known as *Eltanin-*19, it was subsequently singled out as having been 'crucial' to the acceptance of the plate-tectonics explanation of sea-floor spreading. The force of this image depended on the scientists' ability to "illustrate the invisible", accumulating and presenting so much data as a simple pattern or graph.³⁶

A single image now acquired cruciality not only through acceptance by critics that the image does represent relevant features of the natural world, but also through the techniques that compress the results of much labour into a form that displays the feature highlighted by the new model. The symmetry of patterns either side of the Juan de Fuca ridge reassured critics.

4.1 Enhancing Representations.

Looking now at how the representations changed, it is possible to interpret their development as a process of *enhancement* by the addition of dimensions.

I. From the 2-D anomaly patterns to a simple 3-D representation of the sea floor as alternating strips of polarized basalt: The sea floor was then viewed as a magnetic record of some process whose nature geologists could not yet agree upon. Vine and colleagues proposed a 4-D or process model which, if correct, would explain the patterns in terms of the geological history of the sea floor. An important point about the process model was that it predicted a particular, previously neglected feature of magnetometer scans. It treated the 3-D structural representation of sea floor expansion as the consequence of a process, i.e., as a current state of a 4-D representation. Therefore it explained at least some features of the original anomaly maps as consequences of the same process. These two stages are summarised in Table 1, in which dimensions increase from left to right. Each column represents a different order of representational capability. Movement to the right in any row displays dimensional enhancement.

dimensions	2	3	4
I. type of represen-	anomaly map	static model	symbolised theory (images
tation	(pattern)	(structure)	linked by process model)

Table 1. Enhancement by adding dimensions

Acceptance of the process-model on the basis of the profiles plotted from magnetometer scans required three further steps. The first was Vine and Matthew's derivation of the symmetry or mirroring either side of a ridge. Next was the production of a profile that also exhibited mirroring. Finally, there is formal consolidation in which the striping effect is accepted as a necessary consequence of the theory. Mirroring is identified as a special feature of an array of stripes. It is then possible to argue that profiles showing such symmetry are crucial.³⁷ This is illustrated in Table 2 which shows both representational enhancement and formal consolidation. Here as before, each of columns 2-4 hold representations of a different dimensionality. The right-most column contains the derived consequence (whose dimensionality is typically less than that of the process model). Each new *row* contains a new step or move:

II. *derivation* of the possibility of symmetry,

III. *identifying* this in certain maps and profiles which become definitive, and

IV. consolidation, i.e. establishing the original maps and profiles as consequences of the processes postulated in I.

dimensions	2	3	4	derivation
l. representation	anomaly maps profiles	static model	process model	symmetry in striping either side of a ridge
II. new feature				
III. representation	search/generate ano- maly maps and profiles for symmetry			selected features of existing / new maps
IV. representation depicts real world feature	selected anomaly maps and profiles that show symmetry			selected maps and profiles

Table 2. Enhancement and Formal Consolidation

The use of a matrix to represent visual inference achieves two things. First, it enables us to distinguish those aspects of a cognitive process that we do not yet understand from those what we may already understand. In Table 2 a move across a row represents an inference whose cognitive character remains opaque, i.e. beyond the reach of current theories of cognitive science. A move downwards represents an inference that may prove, on further analysis, to involve induction and/or deduction. This is because horizontal moves generate new representations that are stable enough to use in other, less opaque kinds of inference. Table 2 incorporates dimensional enhancement (as rows) and other kinds of inference (as the generation of new rows). The table situates these processes in relation to a larger pattern of inference. The second achievement of the matrix form is that it takes us away from the linearity of a narrative account. Thus it captures certain features of visual inference in a science. This does not yet give us an integrated cognitive, social and historical theory of visualization in science, but I believe it will help.³⁸ I believe that the progression from two- to four dimensions is a general characteristic of the use of drawings, sketches, graphs and more complex visualizations in science. If I am right, then we should be able to find other examples of visual abduction as a process in which images are generated, incorporated into arguments and later established as crucial.

These examples should fit this schema. A second example, drawn from the micro-history of discovery, will illustrate the point.

4.2 Imaging Phenomena.

I now turn to a very different case. However, I believe that this involves the same processes of manipulation and enhancement and that these processes follow an abductive pattern. In his laboratory-based, bench-top explorations of electromagnetism Faraday construed many of his experiments as showing a temporal slice - a 'snapshot' - of the effect of some more complex but hidden, physical process. This work was part of a response to Oersted's momentous discovery that a current-carrying wire has magnetic properties. This is an important instance of the impact of an anomalous new phenomenon on an established paradigm.³⁹ By September of 1821 Faraday and Davy had developed experimental methods of integrating discrete experimental events (or rather, of integrating the images that depict them). They combined discrete images obtained over time into a single geometrical structure and they created a physical structure of sensors with which to record the effects of a single event at different points of space.⁴⁰ A typical procedure involved carefully positioning one or more needles in the region of a wire, connecting the circuit to a battery and observing the effect on the needle(s). Similarly, continuous exploration of the space around the wire would produce many discrete observations of needle positions. Davy and Faraday interpreted these results in terms of a three-dimensional representation of the magnetic effects of the current. A structure of needles arranged in a spiral around the wire gave a three-dimensional magnetic 'snapshot' of the magnetizing effect of the current. Another setup, a horizontal disc with needles arranged around its perimeter, emerged from a set of temporally distinct observations, which it integrates into a single spatial array.

They understood the observable, two-dimensional patterns of magnetised needles and iron filings as spatial or temporal sections of models of processes whose complexity or speed placed them beyond the reach of unaided observation. From an early experiment in which a battery was discharged through a vertical wire passing through a cardboard disk on which either steel needles or iron filings had been arranged, Davy concluded that "as many polar arrangements may be formed as chords can be drawn in circles surrounding the wire; and so far these phenomena agree with [Wollaston's] idea of revolving magnetism"⁴¹ This articulates an inference made from a two-dimensional arrangement to a four-dimensional process (a 'revolving' structure of 'magnetism'). From these 'maps' they hoped to develop structures that could explain effects at every place of action. They would then apply the structural model to the interpretation of other phenomena.

There was no induction here. Observed patterns explained nothing in themselves. However, once they had been identified as features of a process, the patterns could suggest and guide further exploration of structures impossible to observe. These should manifest themselves as other (new) patterns. An important example is Davy's explanation of phenomena observed in an experiment carried out in May of 1821. Assisted by Faraday, Davy passed a current through a vacuum to produce a luminous glow discharge. Davy reported that:

a powerful magnet presented to this arc [luminous] or column, having its pole at a very acute angle to it, the arc, or column, was attracted or repelled with a rotatory motion, or made to revolve by placing the poles in different positions, according to the same law ... as described in my last paper.⁴²

Davy and Faraday construed this process in terms of hidden, real-time (4-D) processes involving 3-D structures.

Faraday later developed this approach with devices to 'extend' his ability to analyze high frequency processes. Another method was to reproduce patterned appearances by means of mechanical simulations. Where he could simulate some aspect of a natural phenomenon by a high speed mechanical process, Faraday took this to be a fair indication as to the nature of that process. Typical simulations were the toothed wooden wheels whose rotation could reproduce apparent rotation of the apparent discs of aquatic animalcules (observed by Leeuwenhoek in 1702, but shown by Faraday in 1831 to be progressive undulations in their *cilia*) and his simulation of the appearance of the surface of a vibrating fluid using a perforated silver plate.⁴³

Faraday's notebooks provide a wealth of information about the interaction of different percepts and emerging representations at any particular time. If the progression from pattern through structure to process characterises Faraday's experimental reasoning, then it should also be found in records that are rich in images, such as the day's work that led to the first working electric rotation motor. By September of 1821 when he returned to the examination of single wire-needle interactions, Faraday had become skilled in making these apparatus-based spatio-temporal transformations.

I have described the background in some detail in order to indicate the skills and experience that Faraday brought to his first independent investigations of electromagnetism in the summer of 1821. I will analyze the work of a single day which led him to construct the first electric motor.⁴⁴ The micro-structure of exploratory work displays the same process of dimensional reduction (whereby selected features are represented visually, as *patterns*), followed by enhancements leading to new, 3-dimensional *structures*, reductions that generate predictions about new phenomena, and finally, consolidation which establishes the derived structures as plausible explanations or realizations of the observed patterns. In this case visualization produced a new *material* artefact (a motor) rather than an explanatory theory about visualized data such as magnetometer scans.

Faraday's notes for experiments of 3 and 4 September 1821 begin with a re-examination of the magnet-wire interactions that he had helped Davy and Wollaston to investigate.⁴⁵ This work used the circular image as an heuristic for subsequent exploration with more complex experimental setups.⁴⁶ To show how, I will recount the steps in his exploration of the interaction of magnets and currents.

According to my analysis of Faraday's notes of his work on the interaction of currents and magnets, Faraday first repeated the observation of the attractive and repulsive effects of the current, paying particular attention to the effect of position. Like Davy and Wollaston, he believed that the magnetism in the region of the wire was somehow structured. This appears to have been the main focus of this investigation. At this stage all he had to go on were the magnetization patterns he and Davy had produced earlier.⁴⁷ The first set of sketches records a more detailed examination of the space around the current. The arrows relate apparent *needle motions* to needle *positions* relative to the wire (see Table 3). The next set superimposes several sets of such observations into a single pair of diagrams. This actually reduces an observed real-world process to a two-dimensional 'map'. It is important to note that Faraday next manipulates objects in the map. The next figure shows the same set

of accumulated observations, but rotated through 90°. In practice it would have been very difficult to observe even one instance of this.

Thus far we have a complex set of observations reduced to a 2D map which is then manipulated (by mental rotation) to create a 3D mental model of a whole set of needle-wire interactions. This is an abduction to something new. There are neither rules nor symbols to express steps by which such a transformation might have been made. Faraday used the 3D representation to infer the possibility of motion in circles.

These moves are shown in Table 3 which uses the same conventions as Table 2: moving horizontally across the matrix denotes adding or removing dimensions, that is, the creation of new representations. I do not regard horizontal moves as abduction. However, a downward move to a new row denotes an abductive inference to a new phenomenon or structure. This is usually represented in terms of images already present in higher rows, i.e. downward moves involve the invention of a new *interpretation* rather than a new image.

dimensions	2	3	4	derivation
I. representation	four positions of magnetic attraction & repulsion	?	complex process which cannot be observed	pattern?
II. initial reduction and enhancement to new feature	"strong attraction repulsion"	viewed from above, in 3-D		symmetry in relationship of position to attractive or repulsive effect
III. re-presentation: second reduction and enhancement	eight positions of attraction & repulsion		"Hence the wire moves in opposite circles"	circulation of wire around magnet is possible
IV. representation depicts real world feature		first experimental setup (paras. 7-8)	inhibited motions obtained	reconfigure the setup

Table 3. Enhancement and Consolidation

Motion in circles is inferred via another mental transformation. Instead of imagining a set of needle positions, Faraday uses a stationary needle as a kind of reference frame. He appears to image how the needle would tend to move if the wire were moved so as to occupy each of the positions shown in the diagram (the wire is vertical and indicated in section, i.e. at right angles to the page). These positions fall on the circumference of a circle, so he infers that the needle would move in a circle.

Faraday constructed an image of circular motion by the process I have called *dimensional enhancement*. He used the circle as an heuristic for further constructive work which realized continuous motion of a wire as a phenomenon in the world. In the next stage of investigation the problem was to realise the right set of physical constraints. This was far from straightforward. Faraday made several attempts before he succeeded in constructing a device that could realize continuous circular motion as a phenomenon in the world.

His record shows that one of his first set ups, consisting of a fixed magnet and a wire suspended by flotation so as to be capable of motion, produced lateral or 'side to side' motion of the wire. This reproduced a subset of the original array of needle-wire motions. Faraday bent the wire into a crank, attempting to 'push' it by repositioning the magnet. Here again, his technique was to accumulate many discrete actions (magnetpushes) into a single process (motion). The results are summarized by the two images of circles surrounded by the letters 'N' and 'S' which indicate the positions of north and south poles. (He noted that the magnet was held at right angles to the wire). Although motion of the wire (or 'crank') depended on human intervention it showed him what had been wrong with his earlier setup. The magnet should be parallel to the current. The wire had to be constrained yet free to move around, but Faraday now realised that wire and magnet could be aligned along a common axis along which the current could pass. This showed him where to position the magnet. Motion of the wire around the stationary pole would then follow.

- Adding Dimensions
- Cunspecified inference to new process/phenomenon
- Abduction

Faraday first added dimensionality (moving from left to right in **Table 4**) then systematically removed it in order to derive the new phenomenological consequences (moving downwards in the right hand column in **table 4**), to effects identical to the original 'source' phenomena and also to new possibilities for experimentation.

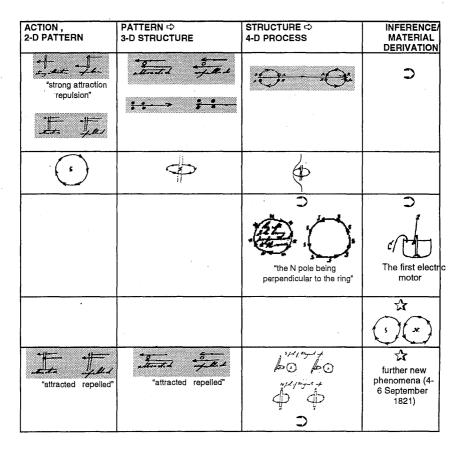


Table 4. Construction and Generation

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This was the third major change to the configuration of his apparatus recorded in his notes. He recorded this inference in words and sketched its outcome in the margin of his notebook. It is represented in Table 4 by the two circles, each with a single pole indicated at its centre. Faraday then made a schematic drawing of this configuration. By the conventions of these tables, it appears in Table 4 as a *derivation*. This produced in reality the hypothetical motions derived in Table 3. The motions sketched in (4d) could now be reconstrued as tendencies to continuous motion, constrained by the physical setup.

The images of circles represent possibilities elicited from a closely observed and manipulated world. The prototype motor constructed at the end of the day *realises* an arrangement of elements of these images, and it reproduces the circular images directly and materially as the circular motion of the tip of the wire suspended in the dish of mercury. So, the circles Faraday drew at 4g (paragraph 13 of his record) are quite different from the ones he had drawn earlier, at para. 6: — they describe an actual effect.

I have used the two dimensions of these tables to bring out the role of the imaging of phenomena at each stage of the process as recorded. The grids display the importance of dimensionality in thinking and how that is woven into a process involving several kinds of inference. As explained earlier, a move from left to right involves enhancement of a representation by adding dimensionality. The fine-grained view afforded by Faraday's record suggests that each horizontal move involves a complex set of manipulations, but it can offer few further clues about these. Vertical moves are of two types: they involve inferring (or deducing) known effects and results (such as the side-to-side motions and the assisted motions of the crank) as well as new phenomenal possibilities. From his knowledge of the mutuality of electromagnetic action (that in the interaction of a wire and a current both will tend to move), Faraday derived the motion of a pivoted magnet around a fixed wire. This was the first of many derivative effects and an important stimulus for the development of his field theory of magnetism.

5. Abduction Revisited

I set out a simple version of Peirce's abductive scheme in section 2. I

have elaborated the notion of visual abduction using matrix-representations in section 4. I used matrices containing both textual and graphical representations to emphasise two points about abduction: first, that it is not necessarily a propositional or symbolic process, and second, that it is possible to characterise abduction more precisely than Kuhn or Hanson attempted to do. We can now elaborate the abductive scheme in terms of these examples, in Table 5.

SCHEME

EXAMPLES (see text)

Abduction

1. Initial An event or action \mathbf{A} (novel or anomalous result)

2. Abduction

Abduce an hypothesis, model or theory H to explain A. Often several (n) models are proposed.

3. Experimental Tests.

Does H imply A?
Test by creating conditions C in which H obtains.
IF TESTS are 'negative': move back to position 2
try another hypothesis, or
re-examine original observation A.
IF TESTS are 'positive': move from position 2 where H could explain A, to the claim that
H probably does explain A.

Consolidation

4. Formal reconstruction (as a Deduction) Reconstruction of steps 1, 2 and 3 to show that A is entailed by H. Result: Deduction: H entails A.

Table 5. Abductive cycles

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6. Creativity and Rationality

Having defined a role for abduction I want to return briefly to the third theme of this volume, *rationality*. Philosophical discussions tend to mean one of two things by 'rational'. Rational can mean conformity to rules of reasoning that are logical (where the very structure of thinking is assumed to be logical) or conformity to method as a set of procedures that obey some logic (where the very method of science is assumed to obey some logic). A broader characterisation includes many other criteria, such as explanatory power, predictive success, simplicity, internal consistency and so on.⁴⁸ The broader characterisation is useful because it allows us to specify what makes science rational without denying its links to other systematic processes (such as the use of evidence in law) or identifying it too closely with idealised, algorithmic procedures (such as logical inference).

Here is where the abductive scheme fits in. It *is* a procedure, yet it is not as restrictive as the traditional inductive and deductive models of inference. For example, it allows us to identify insights or moves which introduce a new point of view without having to reconstruct these as inductions or logical derivations on the one hand, or describe them loosely as gestalt switches. The abductive scheme shows where a novel insight or interpretation fits into a larger framework of mental, physical and social activity. It allows us to characterise a rationale that is not restricted to a particular logical form. Of course some will reconstruct the abductive part of the process as a deduction, but the scheme in Table 5 allows us to specify clearly the context of justification as the *reconstructed* version of events. It is a different sort of account with a different purpose.

A further advantage is that the scheme has some explanatory power: if we treat the novel moves as abductions, then quite a lot of discovery becomes intelligible. We can have a (weak) logic of discovery without having to invoke Platonic intuitions to explain the innovations. This is preferable to placing discovery beyond the reach of systematic methods of analysis and representation, or reconstructing discovery as if it had been, implicitly, a series of deductions.

I have argued that an abductive scheme fits both the text-book story of sea-floor spreading and the far more detailed account of Faraday's construction of a new material configuration of wires and magnets. I do

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not claim to have shown how crucial new insights arise, only that an abductive schema pinpoints where they are introduced and shows their relationship to other experiences, beliefs and, most important, other human activities.

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NOTES

- 1. See Gorman (1992), Gooding (1996).
- 2. Rorty (1980), p. 320.
- 3. See Gooding (1992), pp. 70-72.
- 4. Gooding (1993).
- 5. Nevertheless, many of our figures of speech depend on tactile and proprioceptive experience: see Johnson (1987).
- 6. De Mey (1992), pp. 192-198.
- 7. Shelley (1996).
- 8. De Mey argues that context "is the ubiquitous concept that ... explodes the boundaries between *internal* and *external*, between *cognitive* and *social*, and between *self* and *world*" (1992), p. 253.
- 9. Ibid., p. 280 ff.
- 10. Gooding (1992).
- 11. Brown (1991), pp.
- 12. Hanson (1958), pp.
- 13. There are many examples in Wittgenstein (1953), Hanson (1958, 1969).
- 14. Kuhn (1962), p. 150.
- 15. See Hacking (1993), pp. 305-307.
- 16. See Brown (1991a), pp. 125-6.
- 17. Here, the insight that bodies fall in the same times leads on to the view that acceleration rather than weight is the important factor.
- 18. See Brown (1991b) on thought experiments.
- 19. See Norton (1991).
- 20. By this time the graphical representations are established as what the discourse is about, to such an extent that word-pictures can displace visual representations.
- 21. De Mey likens it to the classical notion of common sense as the coordination or combination of qualities as perceived by different senses, noting that with Descartes the *sensus communis* is associated with a common power of judgement that all human beings have (1992, pp. xxi-xxii).

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- 22. Our need to highlight the visual is a symptom of previous neglect, engendered by the pre-eminence of textual modes of argumentation.
- 23. Richard Gregory and Oliver Sacks have popularised the important insight that may work better when combining these than trying to apprehend the world through one. The widely accepted implication for a cognitive theory of science is that we need to enhance our notion of knowledge to accommodate these different sources.
- 24. In 'Picturing Practice: the role of visualisation in scientific inference', Department of Psychology, University of Nottingham, March 1996.
- 25. Such theories have been envisaged, for example, by Giere (1988), De Mey (1992) and Holmes (1992).
- 26. As Holmes argues, recent studies of experimentation tend to "deal impressively with the way an experimenter seeks to "secure assent" while neglecting "the processes that lead an experimenter to believe that he or she has something to communicate to 'the experimental community'" (1992, p. 126).
- 27. For a post-dualist view of scientific practice see Gooding (1990).
- 28. Simon (1981), p. 153. Much work in AI still models learning and creative work on Simon's assumption that changes of representation enable the representation of problems in a form that will make a solution transparent. See, for example, Cox and Brna (1995), Norman (1994).
- 29. De Mey (1992), p. xxii.
- 30. Latour (1990), Gooding (1990).
- 31. Rudwick (1985), p. 48.
- 32. Rudwick (1985), p.45. The possibility of a "geological dynamics", as Whewell called it, was certainly envisaged. However, as the existence of the controversy over the Devonian formations showed, such explanations were premature.
- 33. LeGrand (1990). See also Giere (1988).
- 34. LeGrand calls such representations symbolic maps. They are analogous to the sections or 3-dimensional representations of geological strata, constructed to display by analogy to coastal cliffs, what would be revealed by cutting through strata.
- 35. LeGrand (1990), p. 253.
- 36. As LeGrand points out, there was resistance because the images were indirect representations of the sea floor. Some regarded profiles generated by computer from magnetimeter data as too distant from the real geological structures.
- 37. As LeGrand notes, other profiles were set aside in favour of those that showed symmetry.
- 38. As advocated, for example, by De Mey (1992), Giere (1988), Miller (1984)

and others.

- 39. See Gooding (1990) and Cantor, Gooding and James (1996).
- 40. In Gooding (1990), chapter 2, I describe these as "process structuring" techniques.
- 41. For references see Gooding, Ibid.
- 42. Faraday recorded this in one of his earliest notebooks (see Gooding, ibid., p. 35).
- 43. See Tweney (1992).
- 44. See Gooding (1990, chapters 2 4) for detailed analyses of this work.
- 45. As he explained in his *Historical Sketch* of electromagnetic experiments (see Gooding 1990), Faraday was convinced that Davy and others had missed something important in the complexity of the phenomenon.
- 46. Faraday's experiments of 3 September not an attempt to verify Oersted's: he had already done that with Davy and Wollaston. Moreover, Faraday made much of the ambiguity of some of their verbal descriptions.
- 47. These illustrations are reproduced in Gooding (1990).
- 48. This is Newton-Smith's 'temperate rationalism' (1981, p. 226 ff. and p. 266 ff.). For an historical investigation of many episodes, aimed at producing a broader set of rules governing scientific change see Laudan, R. (1989) in Laudan et. al, eds. (1989).

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