

## EXEMPLIFYING AN INTERNAL REALIST MODEL OF TRUTH

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### 1. Putnam and Informal Logic

From early on, Hilary Putnam's efforts reflected a deeply foundational result, the Löwenheim-Skolem theorem, that formalized the central intuition that governed much of his thinking. This metamathematical result supports the indeterminacy of the reference relation between theories and their models.<sup>1</sup> As this intuition, captured in many ways, was used to support his many and varied philosophical interests, his concern with formal languages and formal models of, particularly, scientific theories and explanations decreased.<sup>2</sup> In place of metamathematics, Putnam offered various informal and quasi-formal arguments and constructions showing the limits of logical models as a challenge to, among other things, metaphysical realism. This yielded his notion of internal realism.<sup>3</sup>

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<sup>1</sup> Publication dates of earliest works considered below, Putnam, 1978 and 1983, offer a misleading chronology. The discussion of Tarski dates to 1976. The lecture in which the Löwenheim-Skolem issue is raised occurred in 1977. Putnam, 1971 foreshadows his concerns. The notion of reference continues to play an essential role in his later work, e.g. Putnam 1992.

<sup>2</sup> In his latest writing, Putnam, 1994, he discounts the utility of formal apparatus in offering accounts of the sort associated with positivist attempts to display the underlying logic of science in general.

<sup>3</sup> The discussion begins in Putnam, 1978 and 1983. This developed in important directions in Putnam 1981 and 1988. It is the focus of Putnam 1990 and supports, among other things, his arguments against functionalism.

Putnam's move from formal logic parallels the movement within argument theory that is reflected in the embrace of informal logic as the major theoretic perspective for analysis of, what after Douglas Walton we might call reasoned dialogue.<sup>4</sup> The recent movement included many informal logicians with primary interests in fallacy theory and non-formal characterizations of argument structure, as well as argument theorists who drew upon recent work in speech act theory and rhetoric.<sup>5</sup> This movement is characterized by the exploration of argumentation as an interactive process among interlocutors, and the rejection of the argument seen as an atemporal relation between premise and conclusion, as in standard formal logic.

In both Putnam and informal logicians, the movement from mathematical and other formal approaches afforded a flexibility that enabled what people do with language to take center stage. In Putnam the move resulted in rich philosophical discussions, drawing upon the possibility of alternative interpretations to argue against the adequacy of formal theories of reference in relation to realism, functionalism, the role of values in inquiry and many other things. In informal logic it resulted in the careful study of fallacies, and argument structure and function, and a significant concern with the speech acts involved in argument presentation and challenge.<sup>6</sup>

So, one lesson for informal logicians from Putnam might be that a loose and flexible approach to philosophical matters, including logic, is to be preferred to a metamathematical approach. Another, is that metamathematics offers powerful metaphors that enable the logic of many interesting conceptual enterprises to be made clearer.

What I hope to display by example is that the first may be misleading, if the second is true. Putnam's work argues against an aspect

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<sup>4</sup> Walton, 1989.

<sup>5</sup> The literatures in these areas are too vast to even indicate. Ralph Johnson, a leader of the movement, offers a characteristic set of concerns and a comprehensive bibliography in Johnson, 1996. A comprehensive introduction to argument theory is van Eemeren et. al. 1996. Freeman, 1991 offers among the most sophisticated structural accounts elaborating Toulmin et. al. 1979.

<sup>6</sup> Walton, who among the informal logicians has the most concern with speech acts, offers a taxonomy of reasoned dialogue through a speech acts focus in Walton, 1999 and elsewhere.

of mathematical thinking that although traditional is not necessary, that is, the traditional concern for uniqueness of the model relationship in terms of which reference is assigned and the truth predicate defined. This is an understandable approach if your guiding intuition comes from classical geometry, with its deep philosophical connection to ontology—a main thread in logical and mathematical thought from Plato to Kant.

More recently, formal thought, especially as informed by possible worlds semantics, points to flexibility in the field of models available for interpretations of a theory. And so the focus moves from the relation to a model, to relations among models. Putnam uses the notion of possible worlds in many ways in his various philosophical excursions, particularly as part of his arguments against formal theories of reference and functionalism.<sup>7</sup> But he eschews metamathematics and does not explore the possibilities of flexible model-theoretic construction as a device for clarifying problems of the sort that he is concerned with. Without formal constructions, now construable in increasingly flexible ways, many crucial questions remain unanswered, in particular, the question of what a positive account of truth that supports a non-relativistic but context sensitive realism based on objectively definable standards of inquiry might look like.

Such an account would serve as an adequate replacement for traditional formal models now seen as inadequate, and characterized by a naïve correspondence metaphor drawing upon Tarskian semantics in an attempt to satisfy the root intuition that for a sentence to be true it must be about the world. And in doing so, such a replacement should have four essential properties. First, it should be based on a salient model of inquiry, powerful enough to have a *prima facie* relation to truth. Second, it should be flexible enough to capture the dialectic of theory change. Third, it should be normatively compelling. And, fourth, it should have

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<sup>7</sup> A typical, and fairly extended use of Kripke is found in Putnam, 1983. Ironically from the point of view here, Putnam uses Kripke's views negatively as an argument against fixing reference in light of the purported necessity of references across all possible worlds. This reflects the universalism derived from mathematical approaches to ontology rather than the constructive contingency with which reference is determined in the model developed here. We require no more than that reference be determined in light of actual models and accept the contingency and the deflationary pragmatism that such a policy entails. We are no longer interested in abstract possibility, but rely on the possibilities that the range of actual scientific models provides.

a level of articulation that permits its underlying structure to be precisely presented for elaboration and critique. My purpose in what follows is to present just such an account.

I see such an account as complementary to the theory of fallacies which does another job completely (offering a negative theory of argument), and a necessary adjunct to any theory of reasoned dialogue, that includes an argument stage where arguments are seen to require logical force. Most informal logicians and argumentation theorists rely whether overtly or tacitly on the standard account of truth drawn from available formal and quasi-formal systems. Putnam's arguments should show the limits of this approach, if it is not already evident from the inability of complex argumentation to be captured by standard formal models. But more important, I see such an account to offer a positive foundation for an inquiry-based account of realism. I see an adequate theory of truth as necessary if internal realism is to withstand being sociologized and relativized.

## 2. Internal Realism and Truth

Putnam's discussion begins with the tension between realist and verificationist theories of truth. The first is typified by Tarski and the second by philosophers of science who saw truth as the outcome of a process of confirmation (Putnam, 1978, Lectures I and II). In a summative introduction he points to his purposes: 'what Tarski has done is to give us a perfectly correct account of the formal logic of the concept 'true.' But the formal logic of the concept is not all there is to the notion of truth.' (Putnam, 1978, p. 4). What he adds to the formal account leads Putnam to the wide range of philosophical issues that characterize his work. But a governing insight, taken from the verificationist account, pervades much of what followed. The strategy of his approach was to argue against metaphysical realism, and then to move verification from the individual to the social sphere: 'I urged that we accept a species of 'verificationist' semantics. (Though not in the sense of the verificationist theory of *meaning*- for ...'meaning' is not just a function of what goes on 'in our heads', but also of *reference*, and reference is determined by *social* practices and by actual physical paradigms, and not just by what goes on inside any individual speaker.)' (op. cit., p. 129, emphasis in

original).<sup>8</sup> What speakers do is provide reference, and in so doing determine the conditions for truth. And so an understanding of reference and truth is to be found within successful inquiry. This is especially crucial for Putnam's later understanding of what goes on within a process of inquiry, and is reflected in his concern for history and development. (Putnam, 1981)

The Löwenheim-Skolem theorem states that a satisfiable first-order theory (in a countable language) has a countable model. This includes theories of non-countable (non-denumerable) mathematical entities, such as the real numbers. Putnam states the surprising consequence in terms of a consensus in metamathematics: 'All commentators agree that the existence of such models shows that the 'intended' interpretation... is not 'captured' by the formal system.' (Putnam, 1983, p. 3) But then, e.g. the axioms of set theory will have 'no determinate truth-value; they are just true in some intended models and false in others' (ibid. p. 5). How then are intended models to be found? Strikingly and in contrast to metaphysical realism he asserts: 'But the world doesn't pick models or interpret languages. *We* interpret our languages or nothing does.' (ibid. p. 24, emphasis in original) 'Models are not lost noumenal waifs looking for someone to name them; they are constructions within our theory itself, and they have names from birth.' (ibid. p. 25)

The strong connection between truth and models requires a commensurate theory of truth. Without elaborating on the details, Putnam follows Peirce and Dewey in connecting truth with the outcome of practice. But the threat of relativism is all too apparent. Putnam makes a fairly standard move, identifying the conditions for truth with an idealization of epistemological practice: 'A statement is true, in my view, if it would be justified under epistemically ideal conditions.' That is, truth is 'an *idealization* of justification' (ibid. p.84, emphasis in original). Although justification is tensed and relative to a person, truth is not indifferent to reality: 'Just as the objective nature of the environment contributes to the fixing of the reference of terms, so it also contributes to fixing the objective truth conditions.' This view he distinguished from metaphysical realism, calling it 'internal realism'. (ibid. p. 84)

Putnam develops internal realism in a number of places. In *Realism*

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<sup>8</sup> This issue is already an essential concern in Putnam, 1975.

*with a Human Face*, he lays out the following principles under the heading ‘warrant and communal agreement.’ The fundamental desiderata for an inquiry-based internal realism are:

- 1) In ordinary circumstances, there is usually a fact of the matter as to whether the statements people make are warranted or not.
- 2) Whether a statement is warranted or not is independent of whether the majority of one’s cultural peers would *say* it is warranted or unwarranted.
- 3) Our norms and standards of warranted assertability are historical products; they evolve in time.
- 4) Our norms and standards always reflect our interests and values. Our picture of intellectual flourishing is part of, and only makes sense as part of, our picture of human flourishing in general.
- 5) Our norms and standards of *anything*—including warranted assertability—are capable of reform. There are better and worse norms and standards. (Putnam, 1990, p. 21, emphasis in original)

There are essential tensions between 2 and especially 3 and 4. For 2 seems to undermine the identification of standards with a practice, and 3 and 4 engage us deeply with the human project of understanding the world. This connects us immediately with the large project of argumentation theory especially as recently expanded in the work of Johnson and Tindale, both of whom see the connection between the traditional concerns of argumentation theory and large issues of inquiry.<sup>9</sup> But resolving the tension and moving the project forward requires that the social practice of inquiry must be understood and the norms that reflect its informal processes must be articulated. For whatever norms are put forward they have to meet some appropriate demands, drawn from the underlying logic of the practice. And for reasoned dialogue that is a critical inquiry, that requires a sojourn into the realm of truth. Truth in the large sense as the outcome of inquiry undergirds local notions of truth, by setting both an ideal type, and, if my construction is persuasive, affording a useful technical apparatus.

Putnam is concerned with the first of these. For he requires an account of the factual nature of the results of inquiry. This need to ground warrants in the fact of the matter moves Putnam towards

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<sup>9</sup> Johnson, 2000 and Tindale, 2000 are examples, respectively, of the increasing breadth of informal logic theory and the increasing sophistication of rhetoric-based accounts relevant to understanding inquiry.

consideration of an appropriate notion of truth. Putnam points us from the discussion of internal realism in *Realism with a Human Face* to the discussion of truth in *Reason, Truth and History*. In his discussion of 'Two philosophical perspectives,' Putnam rejects metaphysical or externalist realism and with it a 'God's-Eye' view of truth. (Putnam, 1981, p. 50) He also rejects relativism, and supports his conviction in respect of internal realism by offering a restatement of the conditions for an adequate theory of truth. On this view, as elsewhere, the theory of truth sets up ideal conditions. These are articulated in terms of two demands: '1) that truth is independent of justification here and now, but not independent of *all* justification. To claim a statement is true is to claim it could be justified. 2) truth is expected to be stable or 'convergent'; if both a statement and its negation could be 'justified,' even if conditions were as ideal as one could hope to make them, there is no sense in thinking of the statement as *having* a truth-value.' (op. cit., p. 56, emphasis in original) Truth, like reference, is not external to our theorizing but it grows within it.

Putnam is at pains to distinguish his view from relativism and to extend his view beyond mere rational acceptability. (op. cit., p. 55) In places he offers an idealized version of a notion of acceptability strong enough to give us the parameters of a notion of truth. Although the position is well defended and in my opinion substantially correct, Putnam offers little clue as to what the details of an explicit theory of truth in support of internal realism would look like. Putnam's philosophy moves in rather different directions as the consequences of this insight are unfolded: philosophy of mind, the relationship of fact to value, reason in historical setting, etc. His philosophical career moves beyond the metamathematical insight that grounds his work into fruitful areas of deep philosophical insight. This leaves the metamathematical metaphor unexplored and its riches untapped.

The Löwenheim-Skolem theorem requires that metamathematical perspectives move from models to model choices. That is to say, we cannot depend on metamathematics to decide, in Quine's classic epigram, what the values of our variables are. So if to exist is to be the value of a variable, then questions of existence are crucially *aposteriori* in the sense that existence claims require a selection among logically possible models. This, to be rational, requires that the selection be warranted. Inquiry procedures fix this process and justify, in terms of their

pragmatic success, choices made.

But how is the logician to understand this process? Putnam along with many argument theorists relies on the notion of an ideal epistemic community (at the limit a community of one, ideal observer theory) drawing upon the analogous approach in meta-ethics, and with much the same success. Although the notion of an ideal is clearly useful and in some sense correct, without some sense of what warrants the standards to which such an ideal community would appeal truth collapses into acceptability.

For an ideal community account to have any strength it must include some hint of the structure of the standards in light of which the ideal community operates. In ethics, where epistemology may be taken for granted, it is sufficient to stipulate that the ideal community has perfect access to empirical information and reasons perfectly. This leaves the ethical core exposed. But in epistemology to assume that the community has such ideal warrants is just to bypass the crucial epistemological issues. For this we need an account of the sorts of standards that determine perfect application of epistemological principles. Relevant to our concerns here is the contour of the theory of truth and inference that such an ideal community implements in its deliberations.

In what direction do we look for such an account? Informal logic and speech act theories are essential in order to articulate aspects of the normative core of inquiry and other truth seeking endeavors. But neither offers a foundational structure comparable to the model of theoretic elaboration with which Putnam begins: that is metamathematics. And as importantly, the contexts within which these theories are frequently drawn, informal argument, do not offer the foundation in effective practice that affords a strong empirical paradigm from which the theory of truth is to be drawn.

Metamathematical description offers a powerful synoptic metaphor for complex ideational constructs. Like poetry, metamathematics captures the essence of a phenomenon by offering a clear and articulatable sense of its essential relations and underlying structure. Like poetry it gives brief but articulatable images of phenomena that are almost entirely uncharacterizable in other ways. But, like poetry, the power is in the ability to unpack the metaphor. Powerful metamathematical metaphors, like Tarski's theory of truth, permit of indefinite elaboration as the power of the insights are seen by the rich technical literature they engender,

developing the consequences of the view, governed by a clear and relatively well understood yet flexible evaluative process. This sets a very high standard for logical theory. Intuitions, so it seems, need to be cashed out in a fashion that yields perspicuous logical relations, and in a language with well-defined procedures within which implications can be drawn for elaboration and criticism. The problem indicated by Löwenheim-Skolem, is that the standard apparatus underdetermines relevant choices essential for articulating central concepts. This prompts a brief excursion into the standard model.

### 3. A Review of Logic, Inquiry and Truth

James Herman Randall, in his classic exposition of Aristotle, offers a complex view of the relationship between truth, logic and inquiry. (Randall, 1960) The *to dioti* —the why of things, connects apparent truths, the *peri ho*, with explanatory frameworks, through the *archai* of demonstration, that serve as *ta prota*, the first things— a true foundation for apparent truths. Although Aristotle was more ‘postmodern’ than many of those that work in his tradition —the *archai*, after all, were subject matter specific— the envisioning of *archai* as readily knowable, if not known, reflected a classic and overarching optimism about knowledge. This enabled Aristotle to graft a determinate logic onto the various indeterminancies inherent in much of inquiry.

As Randall puts it, “Science’ *episteme* is systematized, ‘formalized’ reasoning; it is demonstration, *apodexsis*, from *archai* ... [it] operates through language, *logos*; through using language, *logismos*, in a certain connected fashion, through *sylogismos*’ (Randall, p. 46) *Sylogismos* points back to the basic constraint on *nous* that it see beyond the accidental and the particular, that it deal with the essential, the *ti esti*, and so syllogism deals with what all of a kind have in common.

Syllogistic reasoning within *episteme* deduces the particular from what all particulars of the kind have in common, and in dialectic looks at the proposed *archai* or *endoxa* through the strongest possible lens: counterexamples as understood in the traditional sense of strict contradictories, systematized, then canonized as the square of opposition.

The focus on *episteme*, on *theoria* places the bar high for those who would propose *archai*. The ‘inductive’ epistemology of concept formation

along with the noetic interpretation of their apperception presupposes that human beings can know reality with an immediacy that seems silly given the course of scientific discovery over the past several centuries. Too much conceptual water has gone under the bridge to think that concepts are to be seen clearly within percepts. Rather, the conceptual frameworks that human beings have elaborated, modified and discarded have been multifarious and extend far beyond the imaginative capabilities of Aristotelian views that take the perceptually presented as representative of underlying realities. Once the enormous difficulty of the task of finding the conceptual apparatus that will undergird a true picture of reality is realized, Aristotle's demand that concepts hold true without exception becomes a serious drag on inquiry. Yet it still prevails, built into the very meaning of logic as used.

Why this is so, is in part because of the power of the next major advance in logical theory. Syllogism, the only completed science as late as Kant, took on a new life when the issues of the foundations of mathematics became the central concern of theorists. The historical connection is not hard to trace; for from Plato, on mathematics was seen as the prototype of knowledge, and its truths a model for the outcome of inquiry. Galileo and Newton linked mathematics to science and so it is no surprise that the logical model, based on the needs of mathematics, retained its grasp on theorists of science as recently as logical empiricism. But there is more to that story, for the enormous advances of the twentieth century took the rudimentary mathematization of syllogism by Boole and others to a theory whose major achievement, completeness, became a model for both what logic is and how it should be understood.

The magnificent achievement of Russell and Tarski offered a model for understanding logical inference and offers a structure open to almost indefinite elaboration — quantification theory — that congruent with much of syllogism, offered a clarity of understanding that surpassed anything dreamt of by centuries of logicians. The Aristotelian core remained, now rethought in terms of extensional interpretations of function symbols that offered a new grounding for the all-or-nothing account of argument built into the square of opposition. The Boolean interpretation of Aristotle's quantifiers retained the high demand that universal claims are to be rejected in light of a single counter-instance, as did the modern semantics of models within which a natural theory of truth was to be found. Mathematizing the clear intuition of correspondence, Tarski's theory of

truth gives the stability needed to yield vast areas of mathematics and even offered some precious, but few, axiomatizations of physical theory. The price was that the truth was relativized to models, yet there was no reason to think that any of the models in use in science were true. This remark requires clarification.

Since the optimistic days in Greece when the theories of inquiry could draw upon few real examples, the claim that *archai* are "noused" from particulars with ease seems a historical curiosity, irrelevant to human inquiry. For the history of human inquiry in the sciences showed that the identification of *archai* is no easy thing. Centuries of scientific advance have shown the utility of all sorts of truism or even downright false models of phenomena. Concepts, and the laws, generalizations and principles that cashed them out into claims, have shown themselves to be mere approximations to a receding reality. As complex connections among concepts, and underlying explanatory frames, have characterized successful inquiry, truth in any absolute sense becomes less of an issue. The issue is, rather, likelihoods, theoretic fecundity, interesting plausibility, etc. The operational concepts behind these —confirmation and disconfirmation— in the once standard philosophical reading (Hempel and the rest), retained the absolutist core that Aristotelian logic exemplifies, amplified by quantification theory. Even Popper saw falsification as instance disconfirmation.

Much work since then has offered a more textured view; I think here of Lakatos (1970) and Laudan (1977). Students of science no longer see the choice as between deductivism —as standardly construed as an account for scientific explanation— and some Feyerabendian allogical procedure that disregards truth. Students of science see, rather, a more nuanced relation between theory building and modification.

Argument theorists and informal logicians should be thrilled at this result for it opens the door for what they do best: the analysis of complex arguments. But not if they are crippled by the very logic that has dominated the discussion so far. Truth, one of the key meta-theoretical underpinnings of logic —along with entailment and relevance— looks rather different when we move from traditional accounts to scientific

practice.<sup>10</sup> Let's look where there is light shining. Let us look to an empirical paradigm for an ideal community.

#### 4. An Internal Realist Model for Truth

If you ask a sane moderately informed person what the world is really made of in just the general sense that Greeks might have asked, the answer is something like "atoms." Let's start there. At the core of modern science stands the periodic table of elements. I take as an assumption that if anything is worth considering true of all of the panoply of modern understanding of the physical world it is that. But why? And what will we learn by changing the paradigm from mathematics to physical chemistry?

The periodic table stands at the center of an amazingly complex joining of theories at levels of analysis from the most ordinary chemical formula in application to industrial needs to the most recondite particle physics. The range of these ordinary things—electrical appliances to bridges—has been interpreted in sequences of models, developed over time, each of these responding to a particular need or area of scientific research. Examples are no more than a listing of scientific understanding of various sorts: the understanding of dyes that prompted organic chemistry in Germany in the late 19th century; the smelting of metals and the improvement of metal kinds, e.g. steel; the work of Faraday and the consequences for the development of electrical apparatus; modern physics and the development of the transistor and the exploration of semi-conductors. This multitude of specific projects, all linked empirically to clear operational concepts, has been unified around two massive theoretic complexes: particle physics and electromagnetic wave theory. But these are only the largest of similar complexes—chemistry, organic chemistry, rigid body mechanics, fluid dynamics. Each of these were independent fields of inquiry, whose power was both evident and evidently increased as they fell together under unifying assumptions.

The deep work in science is to unify theories. The more mundane

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<sup>10</sup> I make the case for entailment and relevance in Weinstein, 1991, and 1995 respectively; also Weinstein 1990.

work is to clarify and extend each of the various applications and clarify and modify existing empirical laws, and this is done in two fashions: 1) by offering better interpretations of empirical and practical understanding as the underlying theories of their structure become clearer, and 2) by strengthening connections between underlying theories so as to move towards a more coherent and comprehensive image of physical reality, as underlying theories are modified and changed. On my reading of physical chemistry, the periodic table is the linchpin, in that it gives us (back to Aristotle again) the key to the basic physical kinds.

We need an image of truth that will support an understanding of inquiry in science. And, surprisingly perhaps, I think the image is just what current argumentation theorists need as well. Since argument is not frozen, logical relations but interactive and ongoing, we need a logic that supports dialectical advance. That is, we need a dynamics of change rather than a statics of proof. We need to see how we reason across different families of considerations, different lines of argument that add plausibility, and affect likelihoods. Arguments are structured arrays of reasons brought forward; that is, argument pervades across an indefinite range of claims and counter-claims. These claims are complex and weigh differently as considerations, depending on how the argument moves. So we need a notion of truth that connects bundles of concerns —lines of argument, and to different degrees.

Back to quantification theory. Quantification theory was developed in order to solve deep problems in the foundations of mathematics. But the standard interpretation of mathematics in arithmetic models proved to be a snare. What was provable is that any theory that had a model, had one in the integers, and arithmetic models became the source for the deepest work in quantification theory. But the naturalness, even ubiquity of a particular model kind does not alter the fact that truth in a model cannot be identified with truth in its ontologically significant sense. All theories have models in the numbers. This does not show that the world is made of numbers, merely that the world is numerable.

Truth in a model is an essential concept. Without it we have no logic. But the identification of truth in a model with truth just reflects the metaphysical and epistemological biases of the tradition in light of the univocal character of mathematics as it was traditionally understood. If I am right, it is not truth in a model that is the central issue for truth, but rather the choice of models that indicate truth. And this cannot be

identified with truth in a model for it requires that models be compared.

To look at it another way, if we replace mathematics with science as the central paradigm from which a logical theory of truth is to be drawn, the identification of truth with truth in a model is severed. For there is no particular model in which scientific theories are proved true. Rather science shows interlocking models connected in weird and wonderful ways. The reduction rules between theories are enormously difficult to find and invariably include all sorts of assumptions not tied to the reduced theory itself. The classic example is the reduction of the gas laws to statistical mechanics. The assumption of equiprobability in regions is just silly as an assumption about real gases, but the assumption permits inferences to be drawn that explain the behavior of gases in a deeply mathematical way, and in a way that gets connected to the developing atomic theory at the time, much to the advantage of theoretical understanding and practical application.

What are the lessons for the theory of truth? We need to get rid of the univocal image of truth, that is, truth within a model, and replace it with the flexibility that modalities both require and support, that is truth across models. We need the metatheoretic subtlety to give mathematical content to likelihoods and plausibilities. A theory of the logic of argument must address the range of moves that ordinary discourse permits as we qualify and modify it in light of countervailing considerations. These can not be squeezed into the Procrustean Bed of all-or-nothing construals of standard logical reasoning.

Formal logic has been captured by Tarski semantics. It offers a clear analogue to the notion of correspondence, but at an enormous price. The power of Tarski semantics —the yield being completeness, that is, all formally valid proofs yield logically true conditionals— requires that the models be extensional, that is, all function symbols in the formal language are definable in terms of regular sets, that is, sets closed under the standard operations of set theory, and definable completely in terms of their extensions. The problem, of course, is that the overwhelming majority of both ordinary and theoretic terms have no obvious extensional definition, and the most interesting functional concepts are intensional (causation, in all of its varieties). I see the essential clue to be the formal solution to modalities (necessity, possibility, and variants such as physical possibility), that is, relationships among worlds as in Kripke semantics. This moves the focus from truth within models, extensionally defined, to

relationships among selected worlds (models). Such relationships may vary widely, each one specific to a particular intensional relationship, as in the analysis of physical causality in terms of a function that maps onto physically possible worlds (worlds consistent with relevant aspects of physical theory). Little can be said about the general restrictions on mappings across worlds, for inter-world relationships, if we take the intuition behind the account of physical causality, are broadly empirico-historical. That is, what makes a world physically possible is relative to the laws of physics interpreted as restrictions on functions across worlds.

The lack of a logical decision procedure —a consequence of the inter-model relations being empirical in the world-historical sense— need not make us despair as to a solution to the problem of truth in principle. For although essential details of the model relationships require an empirical/historical investigation of concepts in use, the functional relations that are concretized in warrants that support entailments and the procedures that determine the relevance of claims and counterclaims, that is, the structure of logical possibilities, can be furnished as constraints on scientific systems.

A solution in principle becomes possible when we look beyond truth in models to truth across models. Within models something very much like the standard interpretation holds, for it enables us to refute our models as we find disconfirming instances.<sup>11</sup> But, across models we need something very different indeed. As mentioned, the account I offer has an affinity with Kripke's solution to the problems of modalities.<sup>12</sup> We look to functional relations among models, and the history of relations over time and in relation to their logical surround. Truth becomes a property of the field. Two basic preliminaries: First, the crucial empirical dimension, for this is science after all. There is a set of privileged models: empirical models of the data. What makes science empirical is a constraint that all models have connections with empirical models. Second, for models at any level short of the highest there may

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<sup>11</sup> I say 'very much like' because I don't want to rule out holding out, even within a model, against disconfirmation. But the clear case of classic contradiction is within models: think of why all men are mortal.

<sup>12</sup> One standard for the adequacy of the formal model developed here would be the ability to define key modal notions such as causation, theoretic entailment, scientific possibility, etc. in its terms.

be found higher level models (a reducing model).<sup>13</sup> So for first level models of the data, these data models are joined through a more theoretical model and on up through reducing models.

Theoretic models take their primary evidentiary force from the empirical models that they join, and then, and more importantly, from the additional empirical models that result from the theoretic joining in excess of the initial empirical base, through models donated from reducing theories. Their power for inquiry derives from the connections among the reducing theories and the possibilities of elaboration that derive from them.

Truthlikeness is defined in terms of considerations such as:

- The increase or decrease in the number and articulation of particular models over time.
- The depth with which any theory is supported by other theories (the height on the vertical of any set of nodes connected by reduction relations at a time, and as a function of time).
- The breadth, the horizontal width which a supporting model is represented in the field of lower level —more empirical— models at a time, and as a function of time.

We need to account for the changing weights assigned to models as they interact. The goal is a metric that correlates with evidence of varying degrees of robustness flowing from different sources.<sup>14</sup> Truthlikeness in complex ways becomes a function of the scientific structure itself. Truth is defined derivatively in terms of optimal outcomes.

To return to our salient example. The periodic table, up pretty high and to the center of the structure envisioned, connects with the vast domain of chemistry, physical and organic, which in association with roughly parallel theoretic clusters, mechanics —statics and dynamics, electro-magnetic wave theory— explains just about everything we do and

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<sup>13</sup> There is no requirement for the highest order models to be univocal (that is the lesson of indeterminacy). Nor that all model chains (paths up the vertical) go particularly high. But since higher order theories deepen the support, we like connections and go as high as we can: in the classic vision of completion, the tip of the Einstein cone—TOES, theories of everything (physical).

<sup>14</sup> A statistic is more reasonable than the linear metric that the formalism indicates, but does not require. Alternatively, rational reconstruction could support metrics, e.g. in building computer models.

can do in the physical world in the last century, and has increased in its explanatory power as individual theories are expanded and refined, and inter-theoretic connections made. Students of each field learn translation procedures to and from observable phenomena —to and from related theories. The connections are often the result of higher order theories. Above the periodic table, there is particle physics, quantum theory, quantum electro-dynamics, and general relativity. Physical practice supports a logical procedure, of drawing appropriately modified inferences within these subject areas. A metamathematical model must support the range of flexibility of such inferences, while yet retaining enough integrity as a model structure so that semantics is indicated and its inferential structure exposed.

There is a logic to the procedure, but it is not the all-or-nothing logic of Aristotle and mathematicians. An argument is not as weak as its weakest link, nor are really weak links much trouble at all. Think of all of the relatively unsupported empirical phenomena that are part of science without having any clearly seen connections to theories. Nobody changes organic chemistry when the latest results on cholesterol in the diet are reported.

Each member of the array supports the others, but they hang separately. That is, particular evidentiary moves affect each model differently. In the immediate neighborhood<sup>15</sup> inquiry affects models in the most intimate way —a near relative of standard logic probably works fine here. But there are relations with other theories, consequences for related theories. A metamathematical model must be flexible enough to allow different strengths of inter-theoretic relationships in order to show how models are isolated or absorbed in the course in inquiry. The shift from a mathematical to a scientific paradigm of truth focuses the theory of inquiry and its underlying logic on change and how it percolates

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<sup>15</sup> The notion of neighborhood looks to a seminal work by Apostel (1961) which specifies a series of approximation relations moving from isomorphic mappings to mappings onto subsets, or onto entities (or relations) within a specified surround. The work of Eberle (1971) offers the logical core. If we think of a grid, a neighborhood would be defined in terms of proximity to a place in the grid. Since relations on the grid may include approximations, nearness affects the degree of structural similarities of mappings across the grid. Whence, near neighbors are affected more by theory change than far neighbors.

through a system.

In what follows I offer a sketch of the formal basis of a model of truth as an example of how a realist position internal to physical chemistry could be supported. If the intuitive power of physical chemistry as a putative ontology is granted, this model becomes a metaphor for how truth works in an area of maximum transparency. How this might be modified in application to areas in which the structure of inquiry is more opaque, remains to be seen.

The details are included in a Technical Appendix. I will indicate the items in the Appendix by the number assigned to it.

The model takes as its object a complex of theory and interpretation, where models of a theory in the fairly standard sense are distinguished from models that result from inter-theoretic relations (what is frequently called 'reduction' in the philosophy of science). All of these are models of the theory, given liberalized assumptions, but the key insight is the indexing of model kinds in terms of their relations across a field.

At the core of the model is an implication relationship, drawn to resist packing by irrelevancies. (Technical Appendix, Part I, 1 & 1.1) Logical problems require the problem of relevance to be decided externally by informed decision, rather than internally as a mark on propositions themselves. This pragmatic turn is supported here in terms of the logical problem to be served, but it is generally supportable as a policy on relevance (Weinstein, 1995).

The deeper pragmatic turn can be seen in the complex construction required to capture the force of actual reductions, which are frequently partial, and approximations of many sorts. (Technical Appendix, Part I, 2 through 2.3) We define a relationship that is open to enormous flexibility, while yet sustaining a clear logical sense of progressivity. However the level of approximation set (by analogy to the priors in Bayesian contexts) is essentially a matter of determinations set a posteriori in light of a reconstruction of the successful practice by inquirers in the field.

The deepest pragmatic turn is how we select from possible models. We do not expect the theory of truth, drawn from scientific, as opposed to mathematical, practice to have full generality. It is contextualized internal to an actual inquiry practice; in the case envisioned, from physical chemistry. So models are actual models in use.

Given this much we can define plausible desiderata relevant to an

internal realist theory of truth. The desiderata unpack intuitions of convergence and progressivity that have been at the heart of realist conceptions of science and science based epistemology and metaphysics since the history of physical science started to support the plausibility of its account of the world vis-à-vis. alternatives.

The basis for the construction elaborated in the Technical Appendix, Part II is a scientific structure defined as an ordered triple,  $TT = \langle T, FF, RR \rangle$ , where:

a)  $T$  is the syntax of  $TT$ , that is a set of sentences that constitute the linguistic statement of  $TT$ . The set  $T$  is closed under some appropriate consequence relation,  $Con$ , where  $Con(T) = \{s: T \vdash_e s\}$ .<sup>16</sup>

b)  $FF$  is a field of sets,  $F$ , such that for all  $F$  in  $FF$ , and  $f$  in  $F$ ,  $f(T') = m$  for some model,  $m$ , where either:

i)  $m \models T$ , or

ii)  $m$  is a near isomorph<sup>17</sup> of some model,  $n$ , and  $n \models T$ .

$FF$  is closed under set-theoretic union: for sets  $X$  and  $Y$ , if  $X$  and  $Y$  are in  $FF$ , so is

$X \cup Y$ .

c)  $RR$  is a field of sets of functions,  $R$ , such that for all  $R$  in  $RR$  and every  $r$  in  $R$ , there is some theory  $T^*$  and  $r$  represents  $T$  in  $T^*$ , in respect of some subset of  $T$ ,  $k(T)$ . We close  $RR$  under set-theoretic union as well.

This enables us to define key notions, articulating the history of  $T$  under the functions in  $F$  and  $R$  of  $FF$  and  $RR$  respectively. An example of the sorts of construction is the basic notion of model chain (Technical Appendix, Part II, 2). The intuition of a model chain permits us to formalize the intuition that a progressive theory expands its domain of application by furnishing theoretic interpretations to an increasingly wide range of phenomena. (Technical Appendix, Part II, 2a) The basic interpretation is the intended model. Thus, theories have epistemic virtue when all models are substantially interpretable in terms of the intended model, or are getting closer to the intended model over time.

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<sup>16</sup> The entailment relation ' $\vdash_e$ ' is precisely defined in Technical Appendix, Part I, 1. Semantic entailment,  $\models$ , reflects an analogous construction.

<sup>17</sup> A near isomorphism is a weakening of the isomorphism relation in definable ways. This requires that models be organized into arrays that permit regions to be specified and metrics to be defined. See foot note 15.

A related, but distinguishable notion, a theory being model progressive, begins with the intuition that theories transcend their domain of applications as they begin as conjectures. This notion defines a sequence of models that capture increasingly many aspects of the theory (Technical Appendix, Part II, 2.1).

The intuition should be clear. A theory's models in the sense of the sets of phenomena to which it is applied must confront the logical expectations the theory provides. That is to say, as the range of application of a theory moves forward in time and across a range of phenomena, the fit between the actual models and the ideal theoretic model defined by the intended model is getting better or is as good as it can get in terms of its articulation.

These constructs, reflecting the model history of a theory T, enable us to evaluate the theory as it stands. By examining the T under RR we add the dimension of theoretic reduction. Similar constructions offer a precise sense of progressiveness under RR. (Technical Appendix, Part II, 3 through 3.4) The key intuition here is that under RR, models are donated from higher-order theories<sup>18</sup> becoming models (or near models) of T while being differentiated from models of T under FF by their derivational history, and the particulars of the RR relation. This enables us to offer essential definitions resulting in a principled ontological commitment in terms of the history of the theory and its relations to other essential theories with which it comports. (Technical Appendix, Part II, 3 through 3.4)

The construction enables us to distinguish particular models and their history across the field, giving us criteria for preference among them. It is this ex post facto selection from among the intended models in light of their history that affords ontological commitment and the related notions of reference and truth. The main contribution of the formal model is how it elucidates the criteria for model choice in terms of the history of the scientific structure, TT, within which a theory sits. That is, we define plausible desiderata, not only upon the theory and its consequences (its models under functions in FF), but in terms of the history of related theories that donate models to the theory under appropriately selected

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<sup>18</sup> Since reduction is irreflexive, asymmetric and transitive, 'higher order' is easily defined in terms of a linear ordering.

reduction relations (functions in RR). It is the structure of the field under these reduction relations, and in particular the breadth and depth of the model chains donated by interlocking reducing theories that determine the epistemic force and ultimately the ontology of the theory. (Technical Appendix, Part II, 4 through 5.31)

The intuition should be clear. A theory, whatever its intended models, takes its ontological commitment in light of how the theory fares in relationship to other theories whose models it incorporates under reduction. That is, we fix reference in light of the facts of the matter, the relevant facts being how the theory is redefined in light of its place in inquiry as inquiry progresses.

It is the awareness on the part of inquirers of the history of success of scientific structures that enables participants in the inquiry to rationally set standards for model choice in terms of plausible criteria, based on successful practice. Crucially, the formal model enables us to look at the history of approximations, and most essentially, goodness-of-fit relations between models donated from above, from reducing theories, and the original interpretations of the theory. Finally, it permits of a natural definition of truth internal to the scientific structure. (Technical Appendix, Part II, 6 through 6.2). Truth is defined in an ideal outcome. Truthlikeness becomes a quantifiable metric as the theories in the structure move towards truth, that is, as the intended model of strong reducing theories substitute as intended models for reduced theories.

The intuition is fairly standard, true theories ramify. The force of the construction is in its logical clarity, and what that permits. The construction displays what one can mean by 'ramify' and indicates how a metric might be defined. The structure could be modeled with any finitary assignment, and consequences drawn. It offers an adequate metaphor for truthlikeness as the outcome of inquiry of the sort found in physical chemistry, a putative candidate for a naturalist ontology. Putnam's work shows that this may be as much as we can hope for, since as things stand the reduction of other essential realms to this ontology may face serious logical barriers.<sup>19</sup>

But it does show that although the possibility of non-standard

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<sup>19</sup> In my initial exploration of scientific structures (Weinstein, 1976) I defined an intuitive notion of incommensurability in light of which boundaries to especially mind-body reduction could be defined.

assignments is logically possible, there are grounds upon which a particular assignment could be warranted. What I hope to suggest is that it is relations between the selected theory and relevant other theories that warrants the choice of referents, and the resulting theory of truth. These theoretic facts of the matter are the objective basis upon which our ontological decisions are made.

Whatever the plausibility of the arguments put forward, the philosophical merit of my view is based on the articulation of the formal model elaborated in the Technical Appendix that follows. The position rests upon the clarity and fecundity of the metamathematical approach that grounds an internalist theory of truth. Whence the paradox of requiring a formal adjunct to the standard informal logic perspective. My claim is that the concern with the dynamics of argument characteristic of informal logicians points to the need for rethinking the implicit mathematics of truth requiring a move from the statics of the traditional account to the dynamic flexibility that the concern with inquiry requires. Shorn of the traditional mathematical model, we look to scientific inquiry for a new metamathematical paradigm.

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## TECHNICAL APPENDIX

### *Part I: Preliminaries*

We begin with a consequence relation, a restricted implication relation. It is constructed after Omer (1970), with an essential modification —the contextualization of the available sentences to a discourse frame. The pragmatic turn, that is, relativization to a discourse frame, saves Omer's construction from inconsistency while preserving its ability to resist manifest irrelevancies of the sort that plagued Hempel and others who worked with the D-N model of explanation.

1. In the following,  $T$  is a set of sentences,  $\{t_1, \dots, t_n\}$ , or alternatively, their conjunction. The *explanandum*,  $s$ , is a sentence. The *explanans*,  $T_c$ , is the longest sequence,  $t_{c1}, \dots, t_{cn}$ , of truth-functional components of  $T$ , or alternatively, their conjunction; and  $\vdash T_c$  iff  $T$ . We say that  $T$  explains  $s$ , in symbols,  $T \vdash_e s$ , just when:

- a)  $T_c$  implies  $s$ ,
- b)  $T_c$  does not imply not- $s$ ,
- c) for some  $t_{ci}$  in  $T_c$ ,  $t_{ci}$  is a nomic generalization,
- d) for any  $t_{ci}$  in  $T_c$ , neither  $t_{ci}$  implies  $s$ , nor  $s$  implies  $t_{ci}$ ,
- e) there is no sequence of sentences  $r_1, \dots, r_k$ , available within the set of sentences accepted by the discourse community that accepts  $T$ , such that, for some sequence of  $t_{c1}, \dots, t_{cj}$  in  $T_c$ 
  - i)  $t_{c1}, \dots, t_{cj}$  implies  $r_1 \& \dots \& r_k$ ,
  - ii)  $r_1, \dots, r_k$  does not imply  $t_{c1} \& \dots \& t_{cj}$ ,
  - iii) upon replacing  $t_{c1}, \dots, t_{cj}$  in  $T_c$  by  $r_1, \dots, r_k$ , in symbols  $T_{cr}$ ,  $T_{cr}$  implies  $s$ .

(Please note: all indices, asterisks etc. are written on the line).  
 A word about condition e). In Omer's construction the phrase "it is not possible to find" played the role that our phrase, "available within the set of sentences accepted by the discourse community that accepts  $T$ ," plays in the above. Omer's formulation leads to inconsistency, since Craig's Interpolation Lemma guarantees that if a) above is satisfied, e) is violated. Our phrase still blocks the sort of artificial constructions available within formal languages that bedeviled the D-N model. We don't lose much since a theory is transparent to other theories in that the set of its models can be expanded under theoretic reduction.

1.1. Despite the identification of explanation with a particular set of premises, ' $\vdash e$ ' is not limited to a unique *explanandum* (Weinstein, 1976). This is crucial for the constructions to follow. ' $\vdash e$ ' is, however, intransitive. This requires the construction linking theories to be distinguished from simple consequence. This distinguishes explanations within theories from explanations across theories (reductions). It also prompts the crucial distinction between consequences under explanation and consequences under reduction, the heart of Part II to follow.

2. The consequence relation above permits us to speak of models of theories. We now move to draw connections between models of various theories. In the following we will build on a broadly flexible notion of reduction developed by Eberle (1971).

Eberle works with the notion of "representing function," a purely syntactic operator that maps formulas and variables of some theory, one to one, onto formulas and variables of another, and, in addition, preserves identity. His account of reduction is a generalization of the notion of *effective representing function*,  $f$ , which is defined so that for a functor  $K$ , that defines some subset of expressions of theories  $T_1$  and  $T_2$ ,  $f$  represents  $T_2$  in  $T_1$  with respect to  $K$  in such a way that for every expression  $e_2$  of  $T_2$ , if  $e_2$  is in  $(T_2)$  then  $f(e_2)$  is an expression,  $e_1$  in  $K(T_1)$ . The notion of reduction is:  $T_2$  reduces to  $T_1$ , just when there is an effective representing function  $r$ , such that  $r$  represent  $T_2$  in  $T_1$  in respect of the consequences of  $T_2$ .

2.1. Two important properties of representing functions:

2.11. If  $f$  is a representing function defined on all expressions of a theory  $T$ ,  $p$  is a formula of  $T$  and  $p$  is a theorem of logic, then  $f(p)$  is a theorem of logic.

2.12.  $f$  reduces  $T_2$  to  $T_1$  is equivalent to, for every model  $m_1$  of  $T_1$  there exists a model of  $M_2$  of  $T_2$  such that, for every sentence  $s_2$  of  $T_2$ ,  $s_2$  is true in  $M_2$  if and only if  $f(s_2)$  is true in  $M_1$ . And similarly for formulas and assignment,  $m_2$ , of  $M_2$  and corresponding formulas in  $T_1$  (p. 490).

2.2. Eberle characterizes this result as showing that if  $T_2$  reduces to  $T_1$ , for every model of the stronger reducing theory one can find a model of the reduced weaker theory which is "roughly speaking, about as close to being equivalent of the former model as one has a right to expect" (p.492).

2.3. Representing functions permit of a wide range of more limited notions of reduction (K limited in special ways; so for example,  $f$  may represent no more than the observational consequences of  $T_2$  in  $T_1$ ). Further, there is a natural limitation on reduction that permits "relativization" in the sense that e.g. formulas of the reduced theory can be replaced by some other formula that performs the same function within limited contexts (p. 497). We assume a very wide range of relations under representing functions in what follows.

### *Part II: Scientific structures*

1. We define a scientific structure as an ordered triple,  $TT = \langle T, FF, RR \rangle$ , with or without indices, primes, asterisks, etc., where:

a)  $T$  is the syntax of  $TT$ , that is a set of sentences that constitute the linguistic statement of  $TT$ . The set  $T$  is closed under some appropriate consequence relation,  $Con$ , where  $Con(T) = \{s: T \vdash e s\}$ .

b)  $FF$  is a field of sets,  $F$ , such that for all  $F$  in  $FF$ , and  $f$  in  $F$ ,  $f(T') = m$  for some model,  $m$ , where either:

i)  $m \models T$ , or

ii)  $m$  is a near isomorph of some model,  $n$ , and  $n \models T$ .

iii)  $FF$  is closed under set-theoretic union: for sets  $X$  and  $Y$ , if  $X$  and  $Y$  are in  $FF$ , so is  $X \cup Y$ .

By "near isomorph" we mean that  $m$  is in some appropriate approximation to the isomorphism relation. The notion of approximation is parasitic on the availability of definable approximation relations, general in respect of a range of possibilities (See, Apostel, 1961). By "appropriate" we offer the second essential pragmatic turn, that is we intend a level of approximation consistent with the practices of the scientific discourse frame within which  $TT$  is sustained. The notion of appropriateness is a posteriori, and we refrain from any attempt to legislate, a priori, what such appropriate approximations should be.

c)  $RR$  is a field of sets of functions,  $R$ , such that for all  $R$  in  $RR$  and every  $r$  in

R, there is some theory  $T^*$  and  $r$  represents  $T$  in  $T^*$ , in respect of some subset of  $T$ ,  $k(T)$ . We close  $RR$  under set-theoretic union as well.

1.1. With  $\text{Con}(T)$  as  $\{s: T \vdash s\}$ , if  $k(T) = \text{Con}(T)$  then  $T^* \text{red} T$ , for some reduction relation.

1.12. Let  $T^* \text{red} T$ , then for any model  $m^*$  of  $T^*$  there is some model  $m$  of  $T$  such that, for all sentences  $t$  in  $T$  (and some  $t^*$  of  $T^*$ )  $m \models t$  if and only if  $m^* \models r(t)$  where  $r(t) = t^*$  under the assignments of the members of  $R$ .

If  $T^* \text{red} T$ , we add to the set  $F^*$ , functions  $f\#$  such that  $f\#(T) = m^*$ . That is, we include in  $TT$  all functions that define models of  $TT^*$  compatible with some models in  $TT$ . Notice that the extension is consistent if the set of models under  $FF$  are consistent.

1.13. We call a model,  $m^*$ , as in the preceding, a reduction model for  $T$ ; the set  $O$  of models  $m^*$ , such that for functions  $f^*$  in  $F^*$ ;  $f^*(T) = m^*$ , is called the *ontic set* of  $T$ .

1.2. Before we begin a more detailed examination of the elements of  $TT$ , a word about the governing intuition is in order. A scientific structure in the sense of  $TT$  is, first of all, a syntax,  $T$ . We then include a class of possible models (or appropriately approximate models) and a set of reducing theories (or near reducers). What we will be interested in is a realization of  $TT$ , that is to say, a triple  $\langle T, F, R \rangle$ , where  $F$  and  $R$  represent choices from  $FF$  and  $RR$ , respectively. What we look at is the history of realizations, that is, an ordered  $n$ -tuple:  $\langle \langle T, F_1, R_1 \rangle, \dots, \langle T, F_n, R_n \rangle \rangle$  ordered in time.

1.21. The syntax  $T$  is an analytic fiction. There will always be boundary disputes between theories and especially between theories and the higher order theories that reduce them. The simplification permits the picture to be drawn in bold outline.

1.22. We claim that the adequacy of  $TT$  as a scientific structure is a complex function of the set of realizations.

1.23. One additional point. Although the construction of realizations, the choice sets of elements, reflect actual practice within a discourse frame, the set of realizations, and the constructions defined through them, are normative in respect of practice. Central epistemological and ontological concepts, in light of which judgments of the adequacy of  $TT$  will be made, are independent of the judgments of members of the discourse frame. That is,  $TT$  need not be consciously available to users of  $TT$ , nor need they explicitly make epistemological or ontological judgments in light of the set of realizations. The adequacy of meta-judgments in respect of  $TT$  is a function of the plausibility of the normative constructs that the set of realizations permit us to see.

2. We define a *model chain*,  $C$ , for theory,  $T$ , as an ordered  $n$ -tuple  $\langle m_1, \dots, m_n \rangle$ , such that for each  $m_i$  in the chain,  $m_i = \langle d_i, g_i, \rangle$  for some

domain,  $d_i$ , and assignment function,  $g_i$ , and where for each  $d_i$  and  $d_j$  in any  $m_i$ ,  $d_i = d_j$ ; and where for each  $i$  and  $j$ ,  $i > j$ ,  $m_i$  is a later realization (in time) of  $T$  than  $m_j$ .

a) Let  $M$  be an intended model of  $T$ , making sure that  $f(T) = M$  for some  $f$  in  $F$ , and that  $M \models T$ . We then say that  $C$  is a *progressive model chain* if:

i) for every  $m_i$  in  $C$ ,  $m_i$  is isomorphic to  $M$ , or

ii) for most pairs  $m_i, m_j$  in  $C$   $i > j$ ,  $m_i$  is a nearer isomorph to  $M$  than  $m_j$ .

This last condition requires a comment. We cannot assume that all theoretic advances are progressive. Frequently, theories move backwards without being, thereby, rejected. We are looking for a preponderance of evidence or where possible, a statistic. Nor can we define this a priori. What counts as an advance is a judgment in respect of a particular enterprise over time. This is another aspect of the pragmatic turn.

**2.1.** Let  $T'$  be a subtheory of  $T$  in the sense that  $T'$  is the restriction of the relational symbols of  $T$  to some sub-set of these symbols. Let  $f'$  be subset of some  $f$  in  $F$ , in some realization of  $TT$ . Let  $\langle T'1, \dots, T'n \rangle$  be an ordered  $n$ -tuple such that for each  $i, j$   $i > j$ ,  $T'i$  reflects a subset of  $T$  modeled under  $f'$  at some time later than  $T'j$ . We say that  $T$  is *model progressive under  $f'$*  if:

a)  $T'k$  is identical to  $T$  for all indices  $k$ , or

b) the ordered  $n$ -tuple  $\langle T'1, \dots, T'n \rangle$  is well ordered in time by the subset relation.

**2.2.** Let  $\langle C1, \dots, Cn \rangle$  be a well ordering of the progressive model chains of  $TT$ , such that for all  $i, j$   $i > j$ ,  $C_i$  is a later model chain than  $C_j$ .  $TT$  is *model chain progressive* if the  $n$ -tuple  $\langle C1, \dots, Cn \rangle$  is well ordered in time by the subset relation.

**3.** We now turn our attention to the members of  $RR$ . Recall that the members of  $RR$  represent  $T$  in  $T^*$  in respect of some subset of  $T$ ,  $k(T)$ . Let  $\langle k1(T), \dots, kn(T) \rangle$  be an  $n$ -tuple of representations of  $T$  over time, that is, if  $i > j$ , then  $k_i(T)$  is a representation of  $T$  in  $T^*$  at a time later than  $k_j(T)$ . We say that  $TT$  is *reduction progressive* if:

a)  $k(T)$  is identical to  $Con(T)$  for all indices, or

b) the  $n$ -tuple is well ordered by the subset relation.

**3.1.** We call an  $n$ -tuple of theories,  $RC = \langle T1, \dots, Tn \rangle$  a *reduction chain*, and,  $\langle T1, \dots, Tn \rangle$  a *deeper reduction chain* than  $j$ -tuple  $\langle T'1, \dots, T'j \rangle$ , if for all  $i, j$  there is a  $r_i$  in  $R_i$  such that  $r_i$  represents  $T_i$  in  $T_{i+1}$  and similarly for  $T'i$  and further  $T_k$  is identical to  $T'k$  for all  $i \leq k$  and  $k \leq j$  and  $n > j$ .

**3.2.** We call a theory *reduction chain progressive* if  $T$  is a member of a series of reduction chains,  $\langle RC1, \dots, RCn \rangle$  and for each  $RC_{i+1}$ ,  $RC_{i+1}$  is a deeper reduction chain than  $T1$ .

**3.3.**  $T$  is a *branching reducer* if there is a pair (at least)  $T'$  and  $T^*$  such that

there is some  $r'$  and  $r^*$  in  $R'$  and  $R^*$ , respectively, such that  $r'$  represents  $T'$  in  $T$  and  $r^*$  represents  $T^*$  in  $T$  and neither  $T'$  is represented in  $T^*$  nor conversely.

**3.31.**  $B = \langle TT1, TT2, \dots, TTn \rangle = \langle \langle T1, F1, R1 \rangle, \langle T2, F2, R2 \rangle, \dots, \langle Tn, Fn, Rn \rangle \rangle$  is a *reduction branch* of  $TT1$  if  $T1$  is a branching reducer in respect of  $Ti$ , and  $Tj$ ,  $i \geq 2$ ;  $j \geq 3$ .

**3.4.** We say that a branching reducer,  $T$ , is a *progressively branching reducer* if the the  $n$ -tuple of reduction branches  $\langle B1, \dots, Bn \rangle$  is well ordered in time by the subset relation.

**4.** Let  $TT\# = \langle TT1, \dots, TTn \rangle$  be an ordering of scientific structures seriously proposed at a time. Let  $\langle \langle T1, F1, R1 \rangle, \dots, \langle Tn, Fn, Rn \rangle \rangle$  be their respective realizations at a time. We say that a set of models  $M$ ,  $M = \langle m1, m2, \dots, mn \rangle$  is a *persistent model set* if for domains,  $d$ ,

a)  $M = \langle \langle m1 = \langle d1, f1 \rangle, m2 = \langle d2, f2 \rangle, \dots, mn = \langle dn, fn \rangle \rangle$  and for all  $i, j$ ,  $di = dj$ , or

b)  $M$  is a persistent model set in a set of ordered subsets of  $TT\#$ , such that the sequence is well ordered in time by the subset relation.

**4.1.**  $M$  is an ontic set for  $TT\#$  (see 1.13).

**4.2.** We say that an ontic set,  $O$ , is a *favoured ontic set* if:

a)  $O$  is the set of intended models of a theory,  $T$ , standing at the head of a progressive reduction chain. (Notice,  $O$  is thus the ontic set of all of the theories in the chain.)

b) the members of the reduction chain are themselves reduction-progressive.

c)  $T$  is a progressively branching reducer.

**4.21.** Notice that the set consisting of an ontic set and the sets that it generates (the set of sets under the reduction relation), forms a persistent model set.

**4.22.** Ontic sets are not the only persistent model sets. Sets composed of experimental realizations common, in our sense, to a set of theories are also persistent. This seems to capture the insight behind instrumentalism in a fashion analogous to the way ontic sets captures the intuition behind realism.

That is, of course, since the models of  $T$  reflect a crucial empirical dimension. That is, there is a set of empirical models of the data and all other models must have connections with some empirical models. But empirical models are not sufficient.

In addition, for models at any level short of the highest, there may be found higher level models. So for first level models of the data, these models of data are joined through a more theoretical model. Theoretic models take their pragmatic force (utility) first from the empirical models that they join, and then, and more importantly, from the additional empirical models that result from the theoretic joining in excess of the initial empirical base of the models joined. The epistemic force (understanding) is a function of the pragmatic force and the

connections defined on the field.

5. TT is *progressive* if:

- a) TT is model chain progressive
- b) TT is model progressive
- c) TT is reduction progressive.

5.1 We call T a *progressive reducer*, if:

- a) T is reduction chain progressive
- b) T is a progressively branching reducer.

5.2. We say T is a *avored reducer* if:

- a) TT is progressive
- b) T is a progressive reducer.

5.3. T is a most favored reducer if T is a maximally progressive reducer, that is, T is the nth member of a reduction chain such that for all  $T_i$ ,  $i < n$ ,  $T_i$  is a favored reducer. (Notice, T is not reduction progressive, since it stands at the head of the longest reduction chain.)

5.31. The set, O, of ontic models of T is thus a favored ontic set in respect of every T' in the reduction chain.

5.32. If T is a most favored reducer, and O is its favored ontic set then O is the ontology of scientific structure TT.

5.33. A truth predicate for TT can then be constructed in fairly standard Tarskian terms as 's is true' for s in T and T in TT, iff  $O = s$  where O is the ontology of TT.

6. Scientific truth, in general, is defined on scientific structures.

6.1. For the scientific realist, truth is: 's' is true iff  $O = s$  where O is as in 5.33 and TT is as big as can be.

6.11. The more useful notion is: TT-true where TT names a unified perspective.

6.12. If TT is, e.g. physical chemistry, physicalism is the theory that theories with mental predicates will eventually be incorporated into TT. Arguments against physicalism purport to show that this is an impossibility. Functionalist theories purport to show how this might be accomplished.

6.2. Degrees of truthlikeness for T at a time are an additive function of the weights assigned on the field at a time. That is to say, theories are assigned weights as prior probabilities. Their model histories give the posterior probabilities in terms of which truthlikeness of any theoretic node is to be ascertained.

6.21. At any point in its history inquirers can be envisioned as assigning posterior probabilities to T as a function of the outcomes of model chains as they become increasingly apparent through time. These probabilities are assigned as a function of the depth and breadth of model chains, as they are constructed in

light of models of the theory under FF, and especially in terms of donated models under RR.

**6.22.** Different strategies for assigning weights come to mind. The simplest strategy is to weigh each node the same and let posterior probabilities sort things out. Instrumentalists might offer more weight to models of the data; realists to selected reducers. Coherence theorists might give extra weight as we move up the chain of reducers (although setting the priors low for high level theories would do the job as well).

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