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SCEPTICISM AND MATHEMATIZATION: PASCAL AND PEIRCE ON MATHEMATICAL EPISTEMOLOGY

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ABSTRACT

In his Pensées, Pascal (1623-1662) introduced the very influential distinction between the subtle intelligence (*esprit de finesse*) and the geometrical intelligence (*esprit géométrique*). In the first part of the present paper Pascal's distinction is considered by looking at his famous wager argument where Pascal acts as a skeptical philosopher and at the same time as an applied mathematician. This argument employs the *esprit de finesse* in a way that is of fundamental significance for the epistemology of mathematics. This claim will be backed up in the second part of the paper that explores Charles Sanders Peirce's conception of diagrammatic reasoning. Peirce's semiotically inspired epistemology of mathematics brings to the fore the significance of the "old" position of Pascal – one has to face the fundamental problems of application.

1. Introduction

The 17th century combines very different traits of culture. It is inasmuch the century of the Thirty Years' War as the century of the scientific revolution. Hence, it is characterized by a specific mixture – the optimistic connotations of science and its ongoing progress meet a radical skepticism. Contemporary paintings, for instance, often show motifs of vanity. Nothing seemed to be secure anymore – except for

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death. André Glucksmann gives a somewhat flowerily but quite appropriate description in his book with the telling title “Descartes c’est la France”:

> Before, societies had diverse, but strong beliefs. They rallied their scattered members around a great cause crowned as the supreme Good by a shared cult. Against that, modern social life as discovered by the moralists does not group together, but disperses. The preordained path of a soul seeking God is replaced by the individual’s games of chance (...). (Glucksmann 1987, 26, my translation from the French.)

To build a new synthesis out of the diverging traits was taken as a challenge by the great philosophical minds of that epoch. This century saw the rise of physics as a science, based in particular on a fundamental upswing of mathematics, connected with names as Descartes, Galilei, Kepler, Newton, Leibniz, and the Bernoullis. Descartes, for instance, was a central figure for the sciences, for mathematics, and for philosophy in that epoch. (For Glucksmann, he was even France!) And significantly, his fundamental idea for a new philosophy came to his mind in 1619, while he had joined the Bavarian army that was going to war against Prague. This was at the beginning of the Thirty Years’ War and Descartes was no ordinary member of the army, rather an intellectual observer, or to say it in modern terms: an embedded philosopher. He exemplifies how two components – universal doubt on the one side and rationalistic certitude on the other side – can come together and initiate a new philosophy. Descartes can be considered as an antipode to Pascal on whose philosophical conception of mathematics the present paper will focus.

Blaise Pascal was born in 1623 and died only 39 years later. As a person, Pascal united different traits in a nearly paradoxical manner. He held a position of fundamental skepticism, directed against that of Descartes, who was employing fundamental philosophical doubt only to obtain a secure basis for his philosophy. At the same time, however, Pascal contributed path-breaking ideas to applied mathematics.

Pascal proved to be an extremely highly gifted mathematician already as a child. Overstraining his mind and powers is said to have been responsible for a long-lasting illness: nearly none of his adult days was without pain. In 1642, Pascal constructed a calculating machine, called “Pascaline”, that caused a sensation all over Europe: A mechanical automaton seemed to be able to carry out tasks of human intelligence! In 1647, Pascal conducted an experiment concerning the pressure of air. At the Puy de Dôme, he found out that this pressure was considerably lower than in the lowlands. This gave rise to a refutation of the Aristotelean dogma of a horror vacui. Pascal, as mostly, used a sharp polemical style. And he expressed a clear commitment to the experimental research method. “The truth has always primacy, no matter how recently it was discovered,” he said in his essay about the void space. In 1654, Pascal announced a treatise on the calculus of probabilities (*géométrie du hazard*) to the Paris Academy. And in the same year he wrote also on the arithmetical triangle.

In these years, Pascal became an adherer to Jansenism, a kind of revival of Augustine’s doctrines. The monastery of Port Royal became the center of Jansenism, fighting against the Jesuits. The leading figure was Antoine Arnauld who, together with Pierre Nicole, wrote one of the most influential books of logic, the so-called “Logic of Port Royal”. Pascal contributed the last chapter to it, which is devoted to probability theory (written around 1658). About 1656, he began to work on an apology of Christianity, addressed to a libertin and gambler, which may not have been completely fictitious, as Pascal had indeed such a friend, namely the duke of Roannez. This project was never finished. The only partially ordered fragments are known as *Pensées* and are acknowledged as the principal work of Pascal. Pascal died at Port Royal in 1662.

While being a strong proponent of the experimental method, Pascal considered the scope of science to be much more limited than did Descartes. Pascal criticized Descartes for being overly optimistic concerning the scientific method, and he often praised Montaigne because of his lessons concerning human ignorance. Consequently, Pascal praised the mathematical method and at the same time reflected philosophically upon the limits of mathematization. (For a more comprehensive account of mathematization as a driving force for mathematics and its limitations, cf. Lenhard and Otte 2005.)

The first part of the present paper will be devoted to Pascal’s seminal contribution to the philosophy of mathematics, especially applied mathematics. His influential distinction between the *esprit géométrique* and the *esprit de finesse* will be considered and further explained by the consideration of an example – the mathematical argument about human ignorance and uncertainty that is the basis for Pascal's famous wager. The interpretation will show that Pascal's argument employs the subtle intelligence in a way that is of fundamental significance for the epistemology of mathematics. In particular, the *esprit de finesse* constitutes an integral part of actually doing mathematics and, therefore, the common view that equates doing mathematics with the mathematical mind (*esprit géométrique*) gives no adequate account. In short, it will be argued in this paper that the applied mode is the basic mode of mathematics.

This claim will be backed up by an investigation of the methodological core of pure mathematics itself, namely the process of deductive reasoning. The second part of the paper explores Charles Sanders Peirce's conception of diagrammatic reasoning. It is based on the insight that all deductive reasoning ultimately rests on the observation of signs. The concluding point will be that a semiotically inspired epistemology of mathematics brings to the fore the significance of the 'old' position of Pascal – one has to face the fundamental problems of application. Thus, based on the views of Pascal and Peirce, the paper's argumentation conducts a kind of pincer-shaped movement against the common view in philosophy that pure mathematics constitutes the basic mode of mathematics. Quite to the contrary, it is argued, applied mathematics is the basic mode of mathematics.

2. The subtle and the geometrical intelligence

In his *Pensées*, Pascal introduces the distinction between the mathematical mind (*esprit géométrique*, geometrical intelligence) and the subtle mind (*esprit de finesse*, subtle intelligence). This distinction is of major importance for epistemology in general, as Ullmo, in the outstanding volume of Le Lionnais on the "great currents of mathematical thought", observed:

> Few writings in our literature have been more influential intellectually than the famous passage in which Pascal distinguished between the geometric intelligence and the subtle intelligence. (Ullmo 1971, 5)

Pascal contrasts the two minds in the following way:

> In the one, the principles are palpable, but removed from ordinary use; so that for want of habit it is difficult to turn one's mind in that direction: (...) But in the intuitive mind the principles are found in common use and are before the eyes of everybody. (...) it is only a question of good eyesight, but it must be good, for the principles are so subtle and so numerous that it is almost impossible but that some escape notice. (512/1, translated by W. F. Trotter)

A short remark on the enumeration according to which the quotations from the *Pensées* are given may be appropriate. The standard reference is Lafuma's who edited the Ouevres complètes in 1963. The second number refers to Brunschvicg's edition, the standard in the decades before. It is remarkable that the given quotation, i.e., the distinction, constituted the very beginning of, thus providing the conceptual framework for Pascal's undertaking. At least it was considered so by the former editor (Brunschvicg). In Lafuma's edition, however, this passage vanishes somewhere in the middle part of the *Pensées*.

One side of Pascal's distinction, the geometrical intelligence, performs a rather clear-cut task, more or less equivalent to the common view on mathematical activity as a whole: it is difficult because of its rigorous method. The other side, the subtle intelligence, seems to form the counterpart of mathematics, although this was not Pascal's intention. Quite on the contrary, for Pascal the problems of the subtle intelligence characterize the methodological core of mathematics and its applications.

Imagine a situation that may potentially be shaped mathematically, or a problem that may be 'mathematized', but a mathematical model has not yet been built. The point is then to find a perspective from which the main features are brought to bear. The mind permanently produces new metaphors or analogies in an associative manner. How can we discern the fruitful ones? How can we conduct the modeling process in an effective way? The first step is to build a model that promises to represent some *essential* features of the situation, ignoring the bulk of other aspects. Thus model-based reasoning enables to observe movement and evolution, i.e., the consequences that follow from hypotheses. In this way facts about and consequences of the assumed hypotheses that could not have been grasped by intuition directly, become visible. Based on growing acquaintance with the model, distinctions may become possible that could never have been based on considerations *a priori*. Usually, this is diagnosed to be a weakness of intuition, and so, in mathematics, analytical reasoning has to play the leading role, it is said.

The mathematical mind is related and also restricted to an ideal layer with clearly defined objects. For instance, consider whether a given equation has a solution? The rules of manipulation are clear, the "principles palpable", but difficult to execute – "removed from ordinary use". Even obligatory education in mathematics over many generations could not influence that substantially.

I want to underline that, according to Pascal, the method of reasoning is universal. In many fields the subtle mind is necessary to apply reasoning appropriately. There is no difference in principle between the mathematical and the subtle mind:

> All mathematicians would then be intuitive if they had clear sight, for they do not reason incorrectly from principles known to them; and intuitive minds would be mathematical if they could turn their eyes to the principles of mathematics to which they are unused. (ibid.)

The point is that both types of reasoning have equal conclusive power, so to say. There is no hierarchy between them, rather they have a complementary relation to one another. But unfortunately, the subtle mind has been interpreted more and more as an anti-pole to mathematics and the sciences. In the course of history, this has led to the differentiation between a more or less mathematical and a subtle kind of rationality. This interpretation, however, does no justice to the intentions of Pascal and is misleading, not only for historical, but also for systematic reasons.

The subtle and the mathematical mind are complementing each other. Moreover, none of them can be omitted, not even in the core of mathematical activity. It is not the case that the *esprit de finesse* would correct the *esprit géométrique*, on the contrary – the problems of applying mathematics involve *both* 'minds'. By the way, this was arguably the opinion of Pascal himself, despite the fact that his distinction has given rise to the exclusive and misleading identification of mathematics with the mathematical mind. Le Lionnais is right in stating:

> Rarely has an opinion been so widely misinterpreted as Pascal's statement concerning what he calls the „Geometric intelligence" and the „subtle intelligence". ... Pascal's work in mathematics and physics is imbued as much with subtlety as with rigor; we may therefore infer that Pascal endowed mathematicians with both kinds of intelligence." (1971, 4)

The mathematical mind infers from principles with necessity. The subtle mind belongs to the realm of judgement. Duhem, for example, relies heavily on Pascal's distinction in his seminal "La théorie physique, son objet et sa structure". He uses it to characterize the science of Britain and France, respectively. However, he does not state a strong interdependence between them. According to Duhem, in modern mathematics the mathematical (Cartesian) mind should rule. At this point, Duhem follows the line of a philosophy of mathematics that recognizes 'pure' mathematics as the 'real' mathematics, at least in methodological respect.

As I have mentioned already, I want to maintain that even mathematics itself needs both 'minds'. In particular, judgement is involved. This claim will be argued for in the following section, considering Pascal's famous wager argument as a case of applied mathematics.

3. Pascal's Wager: a mathematical grip on ignorance and uncertainty

This example stems from Pascal himself, contributing some key ideas to the mathematical theory of probability. In his wager argument, Pascal attempts to show that everyone should conduct a religious life. The point is that he addresses his argument to a gambler. That is, according to the logic of gambling, one should conduct a religious life. Pascal reaches this somewhat surprising conclusion by a mathematical argument about weighing expected gains and losses. The actual argument is, of course, a bit complicated and has attracted a variety of interpretations. Nicolas Rescher, e.g., paraphrases Pascal in the following way:

> When gambling, you people act on the sensible principle of evaluating wagers by blending the chances of an outcome with the

> gain to be realized. Be consistent and do the same in matters of religion. You will then have to agree that no matter how small you deem the chances of God's existence, the infinite reward that will come to the faithful, should He exist, serves to render the gamble of religious commitment worthwhile. (1996, 128)

And Rescher states that "Pascal's famous wager argument is, in fact, an invitation to think about the big issue of life in this world and the next in the manner of a gambler." Rescher's interpretation alludes to Pascal's deep commitment to human ignorance and the limits of rationality. Nevertheless, I think Rescher misses an important dimension of Pascal's argument, for, essentially, it is *not* arguing for the attitude of a gambler in the face of the fundamental restrictions the world poses on human knowledge. Quite the other way round, as, e.g., Béguin (1971) has commented – the listener shall be motivated to comply for *rational* reasons. Hence, there's more to the wager than just an argument for playing a game or motivating an action.

A basic ingredient of the then cultural environment was mentioned at the beginning of the paper: people became very aware of the uncertain and irrational future. Remember the quote from Glucksmann: Throwing the dice denotes more than playing a game – it marks the fundamental condition of human beings. With Pascal's wager, a path-breaking perspective occurred: subjective rationality arguing with a mathematical model of its own situation! In a highly ingenious way, the realm of personal und culturally imprinted convictions was made accessible for mathematical argumentation. This represents an achievement of mathematization *par excellence*.

In his argument, Pascal relied on the infinite potential gain, which renders superfluous an exact knowledge of the probabilities. A chance whatever small would shift the balance. For Pascal, this relation between a finite and an infinite value served as an analogy for that between the finite powers of human rationality and the complexities of the world created by God. In his essay on the geometrical mind, Pascal directly goes from mathematical to moral considerations. In our context here, the main point is that chance, the amorphous counterpart of rationality, could be described using mathematical laws!

Thus Pascal played a pioneering role in two respects: He treated uncertain events as objects and at the same time he initiated the construction of a new mathematical tool – a mathematical concept of probability did not exist at that time. (This concept became widely acknowledged only about 50 years later, while the philosophical interpretation is controversial even today, cf., e.g. Hacking 1975.)

Pascal's argumentation draws on conflicting features. Life is determined by luck and random events and, therefore, the scope of reason is limited. On the other hand, Pascal undertakes the first steps of mathematizing the realm of randomness. This shows that the realm of scientific reasoning is not determined *a priori*, but has to be shaped by practice. To state it in the terms of his own distinction: Pascal prepares the ground for the mathematical mind by *modeling* the situation so that there is something to conclude. This modeling step, however, transformed subjective uncertainty into a wager with comparisons of mathematically defined expectations. Clearly, this was an affair of the *esprit de finesse*. The application of mathematics involves both minds. The construction of a mathematical model, the constitution of ideal objects, is a task of the esprit de finesse. Only if one has in hand a mathematical model of a complex situation, the *esprit géométrique* can enter the stage.

Pascal's distinction was very influential, in part because it was shortened as an opposition between mathematics and the subtle mind. One often finds an identification of mathematics with its deductive modes of reasoning. This seems to be in line with Pascal's distinction, but does not meet his intentions, as we saw. The construction of models and the use of judgement are essential parts of mathematics.

Pascal's ingenious wager argument consists basically in an account that is treating expectations toward an uncertain future and risks mathematically. Why should such a rational construction match with the expectations of humans in the real world? The mathematical argument is convincing only to the extent that one assumes the hypotheses, i.e. the model gives a correct description. This problem is of fundamental importance for the application of mathematics. By the way, Rescher's account of Pascal's wager seems to miss this dimension entirely. Edgar Zilsel has called this problem the application problem ("Das Anwendungsproblem", 1916), first developed in the discussion of probability theory and later extended by Hans Reichenbach, among others, to mathematics in general.

Pascal addressed exactly this problem, trying to work out the convincing force of the wager argument. This is what the really 'subtle' passages of Pascal's text are about: the reader, i.e., the gambler, should be convinced to take the standpoint that is specified by the mathematical model of the situation. In short: the argument has to show the right 'grip' to generate the conviction that the model matches the world.

On first sight, this looks like a specific problem of applied mathematics. This is, however, not the case. One should not identify the mathematical mind with mathematics itself! Nevertheless, this is what happened, and the identification has led to the general opinion that with 'pure' mathematics, mathematics has come to itself. One could indeed argue that applied mathematics is a part of mathematics, and therefore the mentioned problem constitutes a part of the epistemology of mathematics. But, actually, a stronger claim holds: the application problem is fundamental for mathematics in general. As will be shown in the next section that is devoted to C. S. Peirce's analysis of the essence of mathematics, it arises in the inner core of the so-called pure mathematics itself. That is, even in deductive reasoning, the complementary relation of both minds can be found.

4. Peirce's semiotical grip on the essence of mathematics

First of all, deductive reasoning analyses the common outcome of different logical assumptions. The application of very basic rules, like *modus ponens*, involves the observation and recognition of analogies and similarities. Paul Bernays has underlined that to conclude a simple identity may involve highly sophisticated reasoning, because the symbols don't arrange themselves (cf. 1976, 26). In a more Pascalian wording: to conclude an identity may be a difficult task for the *esprit de finesse*. This argument is of rather general significance as Charles Sanders Peirce has shown. He has worked out an epistemology based on semiotics that ascribes mathematics a key role. Initially, he followed the definition of his father Benjamin, a very influential Harvard professor, in stating that mathematics is the science that draws necessary conclusions. But he modified his father's standpoint in a twofold and radical way.

The first transformation is a *pragmatic turn*. It enlarges the conceptual framework and looks at mathematics as an activity oriented toward applications, containing necessary reasoning only as a part. The main question shifts from: *what is the essence of mathematics*, to *what is* *the business of the mathematician?* The second transformation is a semiotic specification. Charles asks further *what is necessary reasoning like?* And his answer consists in the semiotic proposal that each deduction contains elements of perception in an essential way, or proceeds diagrammatically, as he baptized it. The conception of diagrammatic reasoning constitutes a cornerstone of Charles S. Peirce's semiotically inspired epistemology. At the end, it will be argued that diagrammatic reasoning is a conceptually 'sharpened' account of the claim that Pascal's *esprit de finesse* plays a crucial role even in pure mathematics itself (the analytical lion's den).

The subject of this section will be the interpretation of Peirce's 1898 article "The Logic of Mathematics in Relation to Education," bearing the subtitle "Of Mathematics in General." It is contained in the *Collected Papers* (CP), covering paragraphs 553 to 562 of the third volume, according to the usual way of citation: CP 3.552 – 3.562. In this coherent text, Peirce reasons about the essence of mathematics from his philosophical viewpoint. One can find, therefore, characteristic features of the Peircean philosophy in a nutshell.

Commonly, one finds cited only the lines in which Peirce quotes his father's definition. The citation should support the identity of the two viewpoints. But this is not the whole truth, since Charles Sanders transformed his father's definition essentially. For a more detailed comparison of their standpoints, cf. Lenhard (2005). Peirce gives credits to Kant and his conception of mathematics given in the *Critique of Pure Reason*. For Kant, as for Peirce, mathematics played a fundamental role for epistemology and mathematics itself draws very much on constructing diagrams.

> Kant, in the Critique of Pure Reason (Methodology, chapter I, section 1), distinctly rejects the definition of mathematics as the science of quantity. What really distinguishes mathematics, according to him, is not the subject of which it treats, but its method, which consists in studying constructions, or diagrams. That such is its method is unquestionably correct; for, even in algebra, the great purpose which the symbolism subserves is to bring a skeleton representation of the relations concerned in the problem before the mind's eye in a schematic shape, which can be studied much as a geometrical figure is studied. (3.556)

Even this short passage shows clearly how Peirce embeds his semiotic thesis about diagrammatic reasoning in the tradition of Kant. What Kant described as "construction" was interpreted by Peirce as constructing an observable picture or diagram that allows for further empirical analysis.

In Peirce's view, mathematics was no autonomous endeavour – the most important thing to study was the process of application. What is the relation between mathematics and sciences that appeal to it? The answer to this question is pivotal for Peirce's philosophy of mathematics: Defining mathematics requires it to be embedded into the process of scientific research. Only by looking at the whole process does it become possible to acknowledge those aspects of genesis and evolution that were so clear to Peirce.

Now come the decisive paragraphs, introduced by the position of his father who formulated very clearly that mathematics cannot be defined through its objects, but has to be defined "subjectively."

> Of late decades philosophical mathematicians have come to a pretty just understanding of the nature of their own pursuit. I do not know that anybody struck the true note before Benjamin Peirce, who, in 1870, declared mathematics to be "the science which draws necessary conclusions," adding that it must be defined "subjectively" and not "objectively." (3.558)

Doesn't that allude to the common view that pure mathematics would be the 'real' mathematics? Charles quotes his father affirmatively, and therefore one can speak of this definition as the family doctrine. Nevertheless, he modifies it in the following paragraphs and thereby elaborates the original Peircean pragmatic and semiotic shifts. Mathematics relies on models or hypotheses extensively, hence it reasons about an "ideal state of things". This unspectacular statement contains the fundamental insight that the applicability of mathematics depends on the construction of *ideal states* – or expressed in a more modern fashion: it depends on the construction of model worlds. Mathematics makes its assertions only by the use of models and only about models.

> Hence to say that mathematics busies itself in drawing necessary conclusions, and to say that it busies itself with hypotheses, are two statements which the logician perceives come to the same thing. (3.558)

One could exchange Peirces “hypotheses” for “models.” What is the subject matter of “busies itself?” At first sight, it seems as if mathematicians would act solely in the world of models, because it is only there that they can do what characterizes them, namely, to reason with necessity. This self-sufficient attitude of mathematics is closely related to so-called “if-thenism,” and seems to result from the Peircean family doctrine. Exactly at this point, Peirce takes a step that will shape his philosophy of mathematics. He embeds the necessary reasoning, as a part of mathematical activity, into a more global practice of mathematics. This step makes him a close relative to Pascal, claiming that mathematical reasoning is based on *both* intelligences.

The issue does not revolve around *any* hypotheses, but around ones that are judged as adequate for specific reasons, that is, according to criteria of application. In other words, to Peirce, the steps of modeling (that Peirce called “abductive”), of necessary reasoning within a model, and of testing the results inductively by application to the world of phenomena, together constitute one and the same process. This process, in turn, refers not only to a theoretical and methodological practice, but also to a social one. Thus, Charles S. Peirce performs a pragmatic shift in the definition of mathematics. The question of identifying the “essence of mathematics” is transformed into the question of the “business of the mathematician.” In Peirce’s own words:

> A simple way of arriving at a true conception of the mathematician's business is to consider what service it is which he is called in to render in the course of any scientific or other inquiry. Mathematics has always been more or less a trade. An engineer, or a business company (say, an insurance company), or a buyer (say, of land), or a physicist, finds it suits his purpose to ascertain what the necessary consequences of possible facts would be; but the facts are so complicated that he cannot deal with them in his usual way. He calls upon a mathematician and states the question. (3.559)

Now the complications of applied mathematics begin:

> He [the mathematician, J.L.] finds, however, in almost every case that the statement has one inconvenience, and in many cases that it has a second. The first inconvenience is that, though the statement

> may not at first sound very complicated, yet, when it is accurately analyzed, it is found to imply so intricate a condition of things that it far surpasses the power of the mathematician to say with exactitude what its consequence would be. At the same time, it frequently happens that the facts, as stated, are insufficient to answer the question that is put. (3.559)

That is the lesson of the naive model builder: In the process of application, it is neither obvious which facts are relevant, nor how these facts could be molded into a model that is able to allow for a mathematical treatment. There is no unique relation between the field of concrete problems of application and mathematical models.

> Accordingly, the first business of the mathematician, often a most difficult task, is to frame another simpler but quite fictitious problem (supplemented, perhaps, by some supposition), which shall be within his powers, while at the same time it is sufficiently like the problem set before him to answer, well or ill, as a substitute for it. (3.559)

At this point, the process of modeling is getting off the ground. It is worth noting that "to frame another problem" has nothing to do with necessary reasoning, rather it is a question of judgment. Nevertheless, it is "the first business of the mathematician." Peirce is assigning the problems of adequate modeling to mathematics itself! Or, to put it in other words, Pascal's *esprit de finesse* is called in.

Thus he is enlarging the family doctrine considerably. Although mathematics is still not defined through certain objects, the difficulties in handling the objects are still very much involved. In the quotation, Peirce has proposed two criteria:
(i) The question to the model should be practically treatable, "shall be within his powers," and
(ii) the model should be sufficiently adequate, in the sense of applicability, "sufficiently like the problem set before him."
The characteristic tension in applied mathematics is caused by the condition to fulfill (i) and (ii) "at the same time." Both aspects stand in a complementary relation – the more one is easier with one aspect, the more it gets difficult with the other. Normally, neither very simple nor very complex models lead to a solution, because they either have little

significance or are extremely difficult to analyze. The outcome of the process of modeling, never more than preliminarily accepted, is preparing the ground for necessary reasoning, one can say. Thus the latter cannot claim the rank of a definition of mathematics by itself.

> This substituted problem differs also from that which was first set before the mathematician in another respect: namely, that it is highly abstract. All features that have no bearing upon the relations of the premisses to the conclusion are effaced and obliterated. The skeletonization or diagrammatization of the problem serves more purposes than one; but its principal purpose is to strip the significant relations of all disguise. (3.559)

After the pragmatic transformation from *essence* to *business* described above, Peirce poses an even more fundamental question: What is necessary reasoning like? Essentially, his answer is: necessary reasoning proceeds diagrammatically. C. S. Peirce unfolds his argument in the following quotation:

> Kant is entirely right in saying that, in drawing those consequences, the mathematician uses what, in geometry, is called a "construction," or in general a diagram, or visual array of characters or lines. Such a construction is formed according to a precept furnished by the hypothesis. Being formed, the construction is submitted to the scrutiny of observation, and new relations are discovered among its parts, not stated in the precept by which it was formed, and are found, by a little mental experimentation, to be such that they will always be present in such a construction. Thus, the necessary reasoning of mathematics is performed by means of observation and experiment, and its necessary character is due simply to the circumstance that the subject of this observation and experiment is a diagram of our own creation, the conditions of whose being we know all about. (...) All necessary reasoning whatsoever proceeds by constructions; and the only difference between mathematical and philosophical necessary deductions is that the latter are so excessively simple that the construction attracts no attention and is overlooked. The construction exists in the simplest syllogism in Barbara. Why do the logicians like to state a syllogism by writing the major premiss on one line and the minor below it, with letters substituted for the subject and predicates? It is merely because the reasoner has to

> notice that relation between the parts of those premisses which such a diagram brings into prominence. If the reasoner makes use of syllogistic in drawing his conclusion, he has such a diagram or construction in his mind's eye, and observes the result of eliminating the middle term. (3.560)

Peirce is arguing explicitly in favour of the essential similarity of all necessary reasoning, that is, for the thesis that all such reasoning proceeds diagrammatically. The quote gives the principal elements of diagrammatic reasoning, always including “observation and experiment.“ Peirce’s semiotic answer consists in analyzing necessary reasoning as diagrammatic reasoning. This Peircean conception is enjoying growing attention in different areas as semiotics, didactics of mathematics, or artificial intelligence (cf. Stjernfelt 2000; Hoffmann 2004; Chandrasekaran 1995). The pictorial approach seems to me to be especially promising, because the philosophy of mathematics usually suffers from a strong linguistic orientation, which it has inherited from the dominant analytical philosophy of science.

5. Conclusion

The semiotic analysis has led us back to the problem of Pascal again: Even the inner core of pure mathematics is infected with the necessary use of the *esprit de finesse*. Pure mathematics doesn’t differ from applied mathematics in that respect – it is the more recent kind of mathematics and has inherited that trait.

The investigation of the rational power of mathematics has brought to the fore the limits of mathematization: mathematical models cannot be easily imposed on the world. To guarantee that the world can be recognized in the model is the task Pascal was struggling with. As was mentioned, he undertook great efforts to show the convincing force of his argument. This problem is of fundamental importance for the epistemology of mathematics. It is neither solved nor eliminated by Peirce’s semiotic approach, rather posed again more sharply.

Pascal gave an account of mathematics and mathematization in a specific cultural context, namely the skepticism of the 17th century. Thereby he walked a tightrope; proposing a rational argumentation to comply. This mathematization of uncertain events exemplifies that the realm of scientific reasoning has to be determined by practice (a lesson that the philosophy of science had to learn anew). Pascal was convinced that human reason is unable to build its own foundations, which gave rise to his opposition to Descartes. But this conviction did not withhold from pushing the mathematical and scientific approaches as far as he could get them.

Pascal's intellectual venture expressed a fundamental feature of the epistemology of mathematics in a culturally specific way: The limits of mathematization cannot be ignored, even in a more optimistic context. A nice illustration of this statement is offered by a comment on Hermann Weyl by Edgar Zilsel, a critical participant in the Vienna Circle, writing in a time of flourishing philosophy of science. Zilsel argued that a complete argumentation for applicability in logical and mathematical terms would not be feasible, and reflecting upon its limits would have to be part of a philosophical account. Zilsel criticized Weyl's philosophy of mathematics at this point:

> If one would follow through a semiotic epistemology radically, the irrational remainder – concerning reason – would be concentrated on the question, why and under what conditions the construed net of signs can be *applied* on nature, or our experience, respectively. The author mentions this problem only in some methodical guides for research, neither formulating it, nor acknowledging its fundamental importance. (1927, 26, own translation)

This is a Pascalian lesson for the philosophy of mathematics. We saw that the *esprit de finesse* is an integral part of actually doing mathematics and, in this sense, even pure mathematics has to be applied – be it to mathematics itself. Therefore applied mathematics can be conceived of as the basic mode of mathematical reasoning.

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