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# PRINTERS AND ALGEBRAISTS IN MID-16TH CENTURY FRANCE

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#### ABSTRACT

In this paper, we investigate the relationships between two algebraists of the mid-sixteenth century France (Jacques Peletier du Mans and Pierre de La Ramée), and their printers, (Guillaume Cavellat, Jean de Tournes and André Wechel). Both authors published a treatise on algebra at a time when they were involved in a debate concerning French spelling. Did the consideration of these authors concerning symbolism had something to do with their reflections on vernacular language? In Peletier and Ramus's books, the symbolism is specific. Was their choice with regards to the use of symbolism influenced by their printers, or not?

#### 1. Introduction

1.1. The question of the elaboration of the modern « symbolic langage » is commonplace in the history of algebra. One of the first historians to deal with this question was the German philologist and orientalist Georg Heinrich Ferdinand Nesselmann, whose distinction between *rhetorical, syncopated* and *symbolic* algebra has become almost a commonplace depiction. His history of algebraic symbolism is still influential, despite the fact that it raises problems that have been underlined by many historians from the end of the 19th century onwards.<sup>1</sup>

<sup>1</sup> Nesselman's view places Iamblichus, Arabic algebra, Italian abacus algebra and Regiomontanus under rhetorical algebra. The second phase, called syncopated algebra, spans from Diophantus' Arithmetics to European algebra of the sixteenth century; The third phase is symbolic algebra, written with the symbolism we use today. Let's consider the description of the second stage: "One may call the second stage *syncopated algebra*. The exposition is also rhetorical [...] but for certain recurring concepts and operations it uses constant abbreviations instead of complete words. In this stage we find Diophantus and the later Europeans, up to the middle of the seventeenth century, although in his writings Viète had already sown the seed of modern algebra, which nevertheless germinated only sometime after him." (cf. Nesselman, 1842, pp. 301-302). Among the comments about Nesselman's depiction, let's mention those of Tropfke (1933). See also, for an early criticism,

In his second stage, Nesselman mixed authors as different as Diophantus and 16th century European algebraists. In itself, the flowering of mathematical languages in the 16th century is a source of questioning and there is probably no better way to address this question than to attempt *case studies*. In what sense does the symbolism adopted by such or such author appear different from his contemporaries'? In what measure is it original or influenced by previous authors? What are the presumable reasons why this author adopted such or such system? Are these reasons purely mathematical or are they connected with technical considerations such as typographical issues? These are some of the questions we may answer through such case studies.

In this paper, I will focus on the French context and draw attention to a specific topic: The relationships between printers and algebraists. The context is specific: Between 1551 and 1560, several treatises of algebra were published in France by authors such as Jacques Peletier, Pierre Forcadel, Jean Borrel and Pierre de La Ramée (Ramus).

The burst of treatises devoted to algebra in this ten year period has been a source of questioning for historians. Contrary to Italy, there is nothing equivalent to the tradition of abacus textbooks in France.<sup>2</sup> The

Rodet (1881), pp. 69-70. An account on this topic has been recently proposed by Heeffer (2008).

 $<sup>^2</sup>$  Commercial arithmetics in vernacular languages were written in France at the beginning of the 15th century. Among these, let's mention the *Compendy del* 

1550s seem to be the time when algebra suddenly appears in the field of French scholarly mathematics, without relying on a former "local" tradition.<sup>3</sup> Before the publication of Johann Scheubel's *Algebrae compendiosa facilisque descriptio* (1551), few treatises dealing with algebra had been published in France.<sup>4</sup> After 1560, no algebraic treatise was published before Gosselin's *Algebra* (1577).<sup>5</sup> Moreover, these French

*art del algorisme* (Pamiers, c. 1420-1430). In the second half of the 15th century, some arithmetic treatises in French circulated. Between 1475 and 1484, some valuable treatises were written. As for the style, they are close to Italian practical mathematic treatises. Most of these treatises contain no algebra, except Chuquet's *Triparty* (1484), but the *Triparty*, certainly one of the most innovative treatise of the period, had little influence in the first decades of the 16th century, mainly through De La Roche's printed treatise (1520). Finally, the French practical mathematics "tradition" (if ever there was such a tradition) seems poor and it cannot be compared to the Italian abacus tradition. On the Pamiers' manuscript, see Sesiano (1984); On Chuquet, see Spiesser (2006). On the influence of De La Roche and Chuquet, see Heeffer (2012).

<sup>3</sup> On this topic, cf. Van Egmond (1988) and Loget (2012).

<sup>4</sup> *L'arismetique* of Etienne de La Roche, printed in 1520 in Lyon, had little influence during the following decades. Among the mid-sixteenth century algebraists, Borrel is the only one to rely on Etienne de La Roche.

<sup>5</sup> Cf. Gosselin (1577). In this treatise, Gosselin relies on authors such as Pacioli, Cardan, Tartaglia. He also mentions Nuñez and, among his French predecessors, Etienne de La Roche. Gosselin had read Diophantus in the Latin authors, although they rely on previous treatises of algebra (Cardano, Stifel, Scheubel), present some original features. During the 1550s, algebra was introduced in the *curriculum* by some *professeurs royaux*. The publication in Paris of Scheubel, Ramus and Forcadel's treatises is directly connected to this teaching purpose (and that may also be the case of Peletier's treatises). One striking aspect is that French authors seem to have payed special attention to the symbolism and, more generally, to the mathematical language and to the way mathematical reasoning is displayed on the (printed) page.

Moreover, if the French algebraists share a concern about mathematical language, some of them were also involved in a (more lively) controversy about language: the French spelling debate (*réforme de l'orthographe*). Both Peletier and Ramus were involved, at a different level, in this debate that rose in the mid-sixteenth century. As the historian and linguist Nina Catach has shown, printers, along with some authors, played a major role in the standardization of French spelling and typographical syntax.<sup>6</sup> In the case of mathematical language, the issue had a technical dimension in which printers were involved. These facts justify that we investigate the relationships between the authors and their printers and that we endeavour to know if the consideration of the former concerning mathematical language and symbolism has

edition of Xylander (1575). Most of these sources had not been read by the French algebraists of the mid-sixteenth century.

<sup>&</sup>lt;sup>6</sup> Cf. Catach (1968).

something to do with their reflections on vernacular language and if they were or not influenced by their printers.<sup>7</sup>

<sup>&</sup>lt;sup>7</sup> Few historians have raised the question of the role of printing techniques on the evolution of mathematical language during the Renaissance. In his History of Mathematical Notations, Cajori alludes here and there to the role of printers. Many of his remarks deal with typographical errors made by printers, but some highlight the role of printers in the choice of symbolism. For example, he compares the merits of Descartes' notation for exponents in the Geometrie (1637) with his contemporaries' Hérigone, Hume and Stampioen and notes that "From the standpoint of the printer, Hérigone's notation was the simplest, but Descartes' elevated exponent offered certain advantages of interpretation" (1928, p. 346). More recently, G. J. Withrow (1988, p. 266) remarked that the adoption of Roman and Italic types in place of Gothic types (except in Germany) had an effect on the development of mathematical symbolism in that they were "more flexible, particularly in their capacity to combine upper- and lower-case letters". However, not all mathematicians (or printers) immediately seized the opportunity offered by Roman and Italic types and, during the 16th century, some major treatises of algebra were handcrafted using German types. That's the case of Rudolff Coss (1525, 1551). Outside the continent, Recorde's works were also typeset without roman and italic types. Italic types were introduced in France by Sébastien Gryphe in the 1530s and spread troughout Europe in the second half of the 16th century. From then on, most mathematical books were printed using both types.

## 2. The protagonists

The focus on the present paper will be focus on Pierre de La Ramée's and Jacques Peletier's treatises and on the relationships between the two scholars and their printers, Jean de Tournes, Chrétien Wechel and Guillaume Cavellat. Jacques Peletier du Mans' French treatise of algebra was published in 1554 from Jean de Tournes' workshop. Its Latin translation was handcrafted by Guillaume Cavellat in 1560. Pierre de La Ramée's short treatise of algebra was published by André Wechel in 1560.

Apart from the two authors mentioned above, Johann Scheubel's treatise will be considered. As previously mentioned, it was the first treatise of algebra published in France in the 1550s. Two other reasons justify the attention paid to this treatise by this paper: It is the main source for Ramus' *Algebra* and it was printed by Cavellat, who also printed Peletier's Latin treatise of algebra.

2.1 **The Wechel's Workshop.** — Chretien Wechel (†1554), a Belgian native, settled in Paris as a printer in the 1520s. In 1553, his nephew André succeded Chretien as head of the workshop. In 1572, André escaped from the St. Bartholomew's Day massacre and went into exile.

He settled in Frankfurt, where he published, among many other books, Ramus' and Ramist treatises.<sup>8</sup>

Chretien Wechel published all the books where Louis Meigret exposed his spelling reform, such as *Le Męnteur* (1548),<sup>9</sup> *Trętté de la Grammere francoeze* (1550) and *Reponse à l'apologie de Jaques Peletier* (1551). In 1555, the *Pléiade* poets approached André Wechel to have some of their books printed. Jean Antoine de Baïf's books were printed in Wechel's workshop between 1556 and 1558. In these books, Wechel followed Ronsard's spelling for French. As a printer, Chretien Wechel played a role in the French spelling debate.

Ronsard and the *Pléiade* poets may have encountered Ramus in Wechel's workshop, when the *professeur royal* came to meet the printer and started to work with him. In 1555, Wechel published Ramus' *Dialectique* (in French), using modern fonts. In 1557, he published several books by Ramus in which the *Professeur royal* imposed the systematic distinction between *u* and *v* and *i* and *j* for the printing of his works.<sup>10</sup> In

<sup>&</sup>lt;sup>8</sup> Cf. Ewans (1975) and MacLean (2009, chap. 8, "André Wechel at Frankfurt (1572-1581)", pp. 163-226).

<sup>&</sup>lt;sup>9</sup> This translation of *The Liar* of Lucian is, according to Nina Catach (1968, p. 94), the first episode of what has been called the "spelling war" or (as she prefers to say) the "signs war".

<sup>&</sup>lt;sup>10</sup> In 1559 (under one privilège dated June 1557), Wechel publishes *De Moribus* veterum Gallorum, Liber de Caesaris Militia, Grammaticae libri quatuor, Scholae grammaticae et Rudimenta Grammaticae. These two later books, remarks Nina

1559, the capital letters *U* and *J* were produced and added to Wechel's fonts. In 1561, Wechel published Ramus' *Gramere* (1562) in a new French spelling. According to Nina Catach, Ramus' Dialectique (1555) and his Gramere (1562) were typeset thanks to the fonts melted on Wechel's initiative for Louis Meigret's books. Some new types were cut (by Robert Granjon) and added to Wechel's collection of types (cut by Garamond) for printing the Gramere. These facts show that Ramus, during the period he worked along with Wechel, was involved in spelling and typographical issues and worked along with Wechel to impose his views concerning French spelling and typography.<sup>11</sup>

2.2 **Jean de Tournes.** — At the beginning of his career, Jean I de Tournes worked with the famous printer Sébastien Gryphe, then settled as a printer in 1542 in Lyon. He first published the works of the poets

Catach (1968, p. 130), « sont les premiers textes de Ramus où l'on constate, de façon conséquente et définitive, en majuscule comme en minuscule, l'usage du j et du v à Paris ». According to Catach, some types (the capital J and U at least), had been cut for Wechel between April and August 1559.

<sup>&</sup>lt;sup>11</sup> However, the second edition of the *Gramere* (1567) is typeset with standard fonts and the third (1572), which was corrected by Ramus little before he was murdered (and before Wechel's exile in Frankfurt) and the fourth edition (Paris, Denis du Val, 1587) mixed new and old spelling; in the 1587 edition, the printer's foreword indicates that, by that time, no Parisian printer (except himself) was able to reprint Ramus' *Gramere*, in want of the various fonts it required.

Maurice Scève and Louise Labé, from Lyon, as well as some works of Joachim du Bellay. As did Wechel, he played a role in the French spelling debate, mainly through his collaboration with Jacques Peletier, from 1554 to the end of the decade.

In the 1540s, Peletier entered the French spelling debate. His project of a new French spelling, conceived in the 1540s, ended in 1550 with the publication in Poitiers of his *Dialogue de l'ortografe et prononciation francoese départi an deus livres*, along with the *Apologie a Louis Meigret Lionnoez.*<sup>12</sup> At the end of the year 1553 (or at the beginning of 1554), Peletier left Poitiers and traveled to Lyon. He moved to Jean de Tournes' printing house and worked as a *correcteur* in his workshop. Jean de Tournes accepted Peletier's spelling reform and published, under his supervision, Peletier's own books as well as books by some other authors in *orthographe réformé.*<sup>13</sup> During the time he was working with

<sup>12</sup> Peletier lived in Poitiers from the end of the 1540s to 1553. The *Dialogue de l'ortografe et prononciation francoese départi an deus livres* was printed by Jean and Enguilbert de Marnef. The Marnef also handcrafted Peletier's *Arithmetique* in "orthographe réformée". The Marnef house was, according to Nina Catach, one of the most innovative French printers outside Paris. The collaboration between Peletier and Marnef is a first example of the former's commitment to typographical issues.

<sup>13</sup> Among the books published (in French) by Jean de Tournes during the period he collaborated with Peletier, one may mention Louise Labé, *Œuvres* (1556).

Jean de Tournes, some of Peletier's major mathematical and poetical works in French issued from Jean de Tournes' workshop.<sup>14</sup>

2.3 **Guillaume Cavellat.** — Cavellat is a key-figure of the renewal of mathematics in France. In 1547, he settled as a printer close to the *Collège de Cambrai*. Some of the books he published were textbooks that the *lecteurs royaux* were to lecture on and intended for the students of their courses. Some others were textbooks that the *lecteurs royaux* considered being worth publishing. Cavellat specialized in mathematics and natural philosophy textbooks.

In 1551, Cavellat published Johann Scheubel's *Algebrae compendiosa facilisque descriptio*. Before we turn to Ramus' and Peletier's treatises, let's consider this book, the first treatise of algebra printed in Paris. In a foreword to his edition, the Parisian printer Cavellat explained that he published it because of the want of a short treatise of algebra in Paris *academia*. He says he asked some scholars their opinion on Scheubel's book and was answered it was worth printing. One of his advisors was Jean Magnien, a close friend of Pierre de La Ramée and *lecteur royal* of

<sup>&</sup>lt;sup>14</sup> In 1554, Peletier published *L'Aritmetique*, 2d edition (the 1st edition had been published by Marnef, Poitiers, 1552, a third edition was to be be published by Cavellat in 1560) and *L'Algebre*. In 1555, he published *L'Amour des Amours*, his *Art poëtique*, and the second edition of his *Dialogue de l'ortografe et prononciacion francoese départi an deus livres*. Jean de Tournes also published some Latin works by Peletier, such as his famous commentary on Euclid's *Elements: Jacobi Cenomani [...] In Euclidis Elementa commentarius* (1557).

mathematics, and probably the first to teach algebra at the University of Paris: He was, according to Cavellat, "the first [scholar] to refer to algebra in public".<sup>15</sup>

Johann Scheubel, a teacher of mathematics at the University of Tübingen, had first published this treatise in Basel in 1550 under the title *Brevis regularum algebræ descriptio, una cum demonstrationibus geometricis,* along with his commentaries on Euclid's *Elements.*<sup>16</sup> In this quite short treatise (76 pages), Scheubel presented a sort of "classical" and "elementary" algebra. It began with the presentation of the algebraic numbers and of their *numeratio* (*i.e.* the elementary operations on these numbers); He then presented the algorithm to solve equations and illustrated the six canonical types of linear and quadratic equations by some practical examples.

As for the symbolism, Scheubel exposed it at the beginning of his treatise. He first introduced a sign which, when placed to the right of a numeral, signified a "simple" (integer or rational) number. He then introduced the series or powers of the unknown up to the eleventh power and represented them using the "cossist" symbols. However, neither these symbols, nor the cossist nomenclature, were used in the rest of the book. In order not to use many different terms, he explained, he prefered to name the powers *primus, secundus, tertius*, etc., and he represented them by a shortened form *Pri., Se., Ter.*, etc. (fig. 5). As for

<sup>&</sup>lt;sup>15</sup> Cf. Pantin & al. (1986), n. 37, pp. 41-2.

<sup>&</sup>lt;sup>16</sup> Scheubel (1550).

the "simple" number and the radix, they were represented by (resp.) N. and Ra. So, despite the fact that he introduced the cossist symbols, Scheubel neglected these symbols and the cossist names (which he considered too complex) and used mere shortened forms in his computations.

The Paris edition of Scheubel's treatise is close, from a typographical point of view, to the Basel edition (the whole book, however, is typeset in italic instead of roman types).<sup>17</sup> As for the symbols, they are similar in both editions. The sign for the "simple" number and for the first three powers are calligraphy-like symbols:<sup>18</sup>



Fig. 1: Scheubel's symbols in *Algebrae compendiosa facilisque descriptio*, Paris, Cavellat, 1550

<sup>&</sup>lt;sup>17</sup> Scheubel (1551).

<sup>&</sup>lt;sup>18</sup> Indeed, we have evidence of written forms of these symbols in manuscripts. They appear in an "appendix" of the Latin translation of al-Khwârizmî by Robert of Chester, especially the Dresden and Vienna *codices*. Both manuscripts are 15th Century copies (The former may have been in possession of Regiomontanus, the latter of the Leipzig professor Johannes Widmann). Thus, the appendix was probably added to the translation in the late 15th Century and the symbols are not genuine. Cf. Hugues (1989).

The first power symbol may be a shortened form for the Latin word *res.* The second power symbol corresponds here to the Latin *census* and elsewhere to *denarius.*<sup>19</sup> It is also current in printed medical books and pharmacopeias, where it corresponds to the weight unit *dragma.*<sup>20</sup> As for the third power, it may be a ligature *c,s* corresponding to the Latin *cubus.* 

The symbols used in Cavellat's edition of Scheubel's treatise – similar to those used in the original Herwagen edition – stem from handwritten forms used in the German countries in the second half of the 15th century or earlier, before they were cut and included in the printer's fonts. Similar symbols were also used by the Nuremberg printer Johannes Petreius to print Stifel's *Arithmetica integra*. As for those used by Cavellat, we have no indication of their origin, as the description we have of types produced in Paris in this period doesn't

<sup>&</sup>lt;sup>19</sup> According to Høyrup (2010), this sign is derived from the initial letter of the word *zenzo*, usual northern orthography for *censo*.

<sup>&</sup>lt;sup>20</sup> The latin word *dragma* is used either as a monetary or as a weight unit (one third of an ounce). In the first case, it corresponds to the arab *dirham* used by al-Khwârizmî. In the second case, it was currently used in medieval manuscripts and Renaissance printed books, as for example in A.M. Brasavola's *De medicamentis* (1555). It still appears among the medical symbols in Fournier's *Manuel typographique*. Cf. Fournier (1764-66, vol. 2, p. 134).

include algebraic symbols.<sup>21</sup> In any case, Cavellat's edition of Scheubel's treatise is the one in which "cossist" symbols appear in Paris.<sup>22</sup> However, it didn't influence the other French algebraists of the period, as we'll see now by considering Ramus' and Peletier's treatises.

# 3. Authors and books

3.1 **Ramus'** *Algebra* was published by Wechel in 1560, at a time when the *professeur royal d'éloquence et de philosophie* got involved in the teaching of mathematics. There is no evidence that Ramus himself taught algebra, but his booklet, based on Scheubel's *Algebra*, was probably designed for a teaching purpose and intended for students. As some other Ramus textbooks, the *Algebra* was anonymous. But, in opposition to these, it was never re-published under Ramus' name, as if Ramus had given up with the project of writing a full treatise of algebra.<sup>23</sup>

<sup>&</sup>lt;sup>21</sup> Cf. Vervliet (2008). The italic fonts used by Cavellat to typeset the major part of Scheubel's book were probably cut in the mid-1540's.

<sup>22</sup> After Scheubel's treatise, Cavellat published two other books dealing with algebra. A little earlier than Peletier's Latin algebra, he published Pierre Forcadel's treatise of arithmetics. Forcadel intends to give his reader a knowledge of algebra through his account of arithmetics. However Forcadel doesn't use any symbol, except for a *radix* sign ( $\mathbb{R}$ ), here and there. Cf. Forcadel (1556-57).

<sup>&</sup>lt;sup>23</sup> Cf. Loget (2008).

As for the mathematical notations, Ramus' *Algebra* is original. Despite the fact that his treatise is based on Scheubel's *Algebrae compendiosa facilisque descriptio*, Ramus uses neither the cossist symbols nor the shortened forms of algebraic terms used in both versions of Scheubel's book. Instead of these, he uses roman lower-case letters for the powers and the radical symbol (fig. 2).

#### raró tamen supra septimum itur. Sed figuratos satis est primis suorum nominum notis indicari, ut in hac progressione.

1. 2. 4. 8. 16. 32. 64. 128. 256. 512. 1024. 2048. 4096. u. l. q. c. bq. f. qc. bf. tq. cc. fq. tf. bqc. Unitatis tamen notam exprimi necesse non est, cæterorum necesse est cesse notation est cæterorum necesse est cesse est cesse non est cæterorum necesse est cesse est cesse est cesse est cesse est cesse quatuor, decem, centum: at 2 l. 3 c. 4 bq. &c. dices, duo latera, tres cubi, quatuor biquadrati. At cum latus dices, intelligetur quadrati latus, secus si latus alterius figurati, ut cubi, ut biquadrati significes, notabis sic lc. ll, id est latus cubicum, la-

Fig. 2: Pierre de La Ramée, Algebra, Paris, Wechel, 1560, fol. 1r

Why didn't Ramus adopt the uses of his predecessors? To answer this question, we may put forward several hypotheses. The first would be that Wechel did not possess the types needed to typeset an algebra in the "cossist" style. The *Algebra* had been produced hastily, for a student audience and for the need of one specific course. Would this explain the basic typography of the booklet? It is unlikely, given the printer needs few specific symbols to typeset a "cossist"-styled algebra. Furthermore, as we have seen, with Ramus and Wechel, we have an example of a

collaboration between an author and a printer. After Wechel had new types cut and melted for the printing of Meigret's books, this collaboration led to the creation of new types for the printing of the *Gramere* in 1562.<sup>24</sup> For the printing of *Algebra*, Wechel would probably have without hesitation got new types on Ramus' demand, either by having new punches cut or by purchasing the types used by Cavellat earlier.

To explain Ramus' singular choice, the most acceptable hypothesis is that Ramus chose to use lower-case letters by himself, in order (as was also the case for Scheubel when he gave up with cossist symbols) to simplify the language of algebra. Clearly, the use of lower-case letters wasn't an *ad hoc* choice in 1560: Even if his *Algebra* never issued again, Ramus later used the same notation in his *Scholae mathematicae* (1569), published in Basel by Episcopius, to comment on Euclid's book X of the *Elements*. So did Ramus' followers, Salignac and Schöner when they published algebraic treatises based on Ramus' *Algebra.*<sup>25</sup>

**3.2 Peletier's** *Algebre.* — In 1554, Peletier published his *Algebre* in French. As an illustration of Peletier's spelling reform, the whole book is original in its typography. To print *L'Arithmetique* and *L'Algebre* (as well

<sup>&</sup>lt;sup>24</sup> These types were still used in the 1570s by Denis du Val, the successor of Wechel in Paris, for printing some books of J. A. de Baïf, such as the *Etręnes de poezie fransoęze an vęrs mezures* (1574).

<sup>&</sup>lt;sup>25</sup> Cf. Ramus (1569), Salignac (1580) and (Schöner (1586).

as his other books in *orthographe reformée*, such as his *Dialogue* and the *Art poëtique* of 1555), Jean de Tournes used mainly Roman types 12/13 points ("saint-Augustin"), along with some Italic types.<sup>26</sup> Apart from the standard fount of types, Jean de Tournes possessed some special types such as a barred *e* and an *s* with a cedilla, both in Roman and Italic types. These were cut for him by a punchcutter whose name remains unknown and they were necessary to print a book in *orthographe reformée*.

It is commonly accepted that the *Algebre* shows, as for the mathematical notations used, both German and Italian influences. Peletier's main source is Stifel's *Arithmetica integra* (1544). He also relies on Cardano's *Ars magna*. According to Cajori, Peletier's designation of powers and roots is done in the manner of Stifel (but as we shall see, it is not strictly identical from a typographical point of view); He also adopts Stifel's symbolism for the second unknown; On the other hand, like the Italians, Peletier uses p. and m. for "plus" and "minus", instead of the symbols + and -.<sup>27</sup> More generally, despite some refinements (such as his notation for the second unknown, borrowed from Stifel), Peletier's symbolism remains rough from a mathematical point of view.

<sup>&</sup>lt;sup>26</sup> According to Johnson (s.d., p. 53), the Roman types were cut by Garamond and the Italics by Robert Granjon, a follower of Garamond.

<sup>27</sup> Cf. Cajori (1928), vol. 1, p. 172 and Bosmans (1907). As for the + and – signs, Cavellat employed them in Scheubel's *Algebrae compendiosa facilisque descriptio* but Jean de Tournes may have lacked these symbols.

cß

8888

 $x^{11}$ 

Symbol Modern eq. Name x Racine ŖĿ  $\mathbf{x}^2$ S nombre çansique, Çans  $\chi^3$ Cube P  $\chi^4$ Çansiçanse 88 ß  $x^5$ Sursolide, Premier Relat **X**<sup>6</sup> 89 Çansicube  $\chi^7$ bß Second sursolide or Second Relat  $\chi^8$ 888 Cansicansicanse **x**<sup>9</sup> Cubocube 99 çß,  $x^{10}$ 

Troisième sursolide, Troisième Relat

Table 1: Symbols and nomenclature in Peletier, *Algebre*, Lyon, Jean de Tournes, 1554

As Stifel, he introduces four symbols to express the first, second, third and fifth powers. The fourth power and higher-than-fifth powers are expressed by combining (using a "mutiplication rule") several symbols, up to the sixteenth power. Table 1 lists the symbols used by Peletier in 1554 and the French nomenclature he introduces.

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3.2.1 Let's turn to the symbols used in Peletier's *Algebre* for the powers of the unknown (table 1). As for the first power symbol, the sign used by Peletier ( $\overset{\text{RL}}{\longrightarrow}$ ) appears much more common than Stifel's or Scheubel's

<sup>2</sup>C. In mathematics, it came to be used, mainly for the radical sign and occasionally for the first power in the Middle Ages.<sup>28</sup> As for the fifth power symbol, it is also common: Peletier uses a  $\int^{\beta}$  (Italic type). These two symbols are widely spread in Renaissance founts of types,<sup>29</sup> probably because they had been both used since the Middle Ages, notably in medical treatises or pharmacopeias, the former as a shortened form for the latin verb *Recipe* ("Take"), the second for "half".

| 0, 1,                | 2, 3, 4              | f 5,                  | 6, 7,                 | 8,                  | 9,                  | 10,       |
|----------------------|----------------------|-----------------------|-----------------------|---------------------|---------------------|-----------|
| 1, 12,               | ç, 9, ç              | ⊊, ∫ <sup>2</sup> , 9 | gq, bß,               | ççç,                | qq,                 | ςβ,       |
| 1, 2, .              | 4, 8, 1              | ¢, 32,                | 64, 128               | , 256,              | S <sup>12</sup> ,   | 1024,     |
| 11,<br>c/s,<br>2048, | 12,<br>ççq,<br>4096, | 13,<br>dβ,<br>8192,   | 14,<br>çbß,<br>16384, | 15,<br>Aß,<br>32768 | 16<br>525<br>5, 655 | с.<br>36. |

Fig. 3: Jacques Peletier, Algebre, Lyon Jean de Tournes, p. 2

<sup>&</sup>lt;sup>28</sup> Cajori (1928), vol. 1, p. 361, makes mention of uses of the word *radix* for the first power by John of Seville and Gerard of Cremona. As for the symbol  $\mathbb{R}$ , its double use for the first power and square root is encountered in Leonardo of Pisa and Luca Pacioli. In the 16th century, among many others, Cardano uses it in his mathematical as well as in his medical treatises. Its use in medical treatises lasted until the 18th century.

<sup>&</sup>lt;sup>29</sup> The R sign appears in most of Roman types; The double-s sign used for the fifth power exists mainly in Italic types, and Jean de Tournes may not have possessed in his founts the Roman counterpart of the italic type he uses there.

When used in a mathematical context, they are commonly considered as shortened forms for latin words Radix (R: bound letters r,x), sursolidum ( $\beta$ : double s). The signs displayed in the Algebre for the second and third powers are more uncommon. For the second power, Peletier introduces the sign  $\S$  (instead of the sign  $\aleph$  used in Stifel's Arithmetica integra (fig. 4) and in Scheubel's Algebrae compendiosa facilisque descriptio (cf. fig. 5)).<sup>30</sup> One finds no equivalent to this sign in coeval treatises. Should we consider it as bound letters *ç*, *s*? In that case, it would appear as a shortened form of the French name cans (in Peletier's spelling). As for the third power sign,  $\P$ , it is close to the sign used in Stifel's and Scheubel's book ( 🕰, and it is probably a typographical variant of the same sign. To sum up, Peletier bases his symbolism on Stifel's, but, just as his new French spelling requires new types, he has new types introduced to have his treatise of algebra printed. No doubt these new types ( $\mathbf{S}, \mathbf{q}$ ) were cut and melted for printing his Algebre. The other symbols ( $\mathcal{P}$   $\beta$ ) were taken from the (Roman and Italic) founts already in the possession of Jean de Tournes.

<sup>&</sup>lt;sup>30</sup> The design of the symbol is specific: a c with a large barred-cedilla. Jens Høyrup ((2010), p. 48) supposes, considering it looks like the *censo* symbol appearing in the *Aliabraa Argibra* of Dardi of Pisa (written in 1344, extant copies from the late-14th and 15th centuries) and in one Italian manuscript (Modena, bibl. Estense, Ital. 578, c. 1485), that Peletier was acquainted with the Italian manuscript tradition. I do not follow this hypothesis, considering what we now know about Peletier's biography.

Hæc eft igitur progrellio collica, terens denominationes numerorū collicorum. 1.12.13.12.133.18.132.1bß.1333.1ce.128.1cf.1332e. 1dß.13bß.1cf.1333. Et fic deinceps in infinitum.

Fig. 4: Michael Stifel, *Arithmetica integra*, Nuremberg, Petreius, 1544, fol. 235r.

**3.3 Peletier's** *De occulta parte numerorum* —. Let's turn now towards the Latin treatise, published by Cavellat in 1560. Basically, Peletier's system is still based on Stifel's and remains similar as we saw in the 1554 edition of the *Algebre*. However, the symbols used for the powers of the unknown are different from what we have seen before (fig. 6). For the first and fifth power, the signs remain similar as in the French Algebre. The sign for the third power ( $c^{c}$ ) is similar to the one used in Stifel's *Arithmetica integra* and in Scheubel's *Algebrae compendiosa facilisque descriptio*<sup>31</sup>. But for the second power sign, Peletier uses the letter *q* (Roman type), in place of any other symbol he could have used. It was certainly impossible to use there the type designed for the 1554 issue, but Cavellat possessed the  $\frac{3}{5}$  sign in the founts he used for printing

<sup>&</sup>lt;sup>31</sup> However, the design of the sign appears more delicate than the one used in Scheubel's treatise.

Scheubel's *Algebrae compendiosa facilisque descriptio.*<sup>32</sup> The reasons why Peletier chose not to use this symbol are examined hereafter, in the light of his previous choice of 1554.

NVMERATIO. CAPVT I. Haracteres uocabulorum feu appellationum, quíbus ín his regulis numerí naturali quodam ordine proportionis denominantur, sunt, 9, 20, 37, 6, 38, 18, 30%, bl8, 3838.cc, 3868, Tl8, 3856, 866. Signa præterca, + & -CHARACTERVM EXPLICATIO.

Fig 5: Johann Scheubel, *Algebrae compendiosa facilisque descriptio*, Paris, Cavellat, 1551, fol. 1.

### 4. Further reflexions

Just like Ramus, Peletier certainly played a role in the choice of the signs used for printing both issues of his treatise of Algebra. We can only conjecture the reasons why he chose such and such a symbol. Let's

<sup>&</sup>lt;sup>32</sup> I wasn't able to make sure that Scheubel's and Peletier's treatises were typeset with same-sized founts (which is a requirement for using the same *dragma* sign in both books), but it is likely. Isabelle Pantin identifies the Roman and Italic types used in Scheubel's treatise as "R. 118 (?)", "R. 83", "I. 118 (?)", "I. 83" and thoses used in Peletier's treatise as "R. 114", "R. 94 (?)", "R. 83", "R. 66 (?)", "I. 114 (?)", "I. 94". Cf. Pantin & *al.* (1986), n. 37, p. 41-42; n. 161, p. 143.

consider the 1554 sign for the second unknown (\$), the most original in Peletier's set of sign. To explain the design choosen for that sign, one could read the first pages of the *Algebre*, when Peletier presents his nomenclature. After he has given the French names of the powers of the unknown up to the sixteenth power (which he calls *Çansiçanşiçançanşe*) Peletier writes, concerning the later word:

Even if the word sounds rough, it's being significant is enough. And it's quite something to have found a name to such uncommon and rare things.<sup>33</sup>

The reason why Peletier wrote his treatise of algebra in French is well known: He wants to promote French as a scientific language. To achieve this, he coins French names that fit as accurately as possible to the algebraic objects. So, the French names Peletier gives to the powers of the unknown are coined by him in order to correspond to the things named. As for the symbols, his justification may have been similar: As he wants the word to correspond to the thing, he may have wanted the sign to correspond to the name (and, through the name, to the thing). If we were right when seeing in the **S** sign bound letters *ç*,*s*, this very sign is appropriate for the French word *çans*, whereas he maintained the

<sup>&</sup>lt;sup>33</sup> Cf. Peletier (1554), p. 9: "Et encore que le mot semble être rude, il suffit qu'il soit signifiant. Car c'est beaucoup d'avoir trouvé nom à choses si inusitées et si peu pratiquées.". I do not reproduce here Peletier's new spelling.

usual form r,x for radix and s,s for sursolidus. Jean de Tournes certainly had the  $\mathbb{R}$  and  $\beta$  signs at his disposal and they were often used in algebraic treatises: That may have been sufficient enough reason not to have new types cut and melted for *racine* and *sursolide*.

| 4 | 0<br>I<br>I | 1<br>FL<br>2    | 2<br>9<br>4 | 3<br>æ<br>8      | 4<br>99<br>16 | 5<br>Js<br>32 | б<br>qæ<br>64              | 7<br>bß<br>128 | 8<br>999<br>256    | 9<br>AA<br>512  | 10<br>GS<br>1024      |
|---|-------------|-----------------|-------------|------------------|---------------|---------------|----------------------------|----------------|--------------------|-----------------|-----------------------|
|   | 2           | 11<br>cß<br>048 | 9<br>40     | 12<br> qæ<br>096 | 1<br>dj<br>81 | 3<br>B<br>92  | 14<br>9 <i>bfs</i><br>1638 | 4              | 15<br>CLS<br>32768 | 16<br>qq<br>ز ک | 5<br>9 <b>9</b><br>36 |

Fig. 6: Jacques Peletier, *De occulta parte numerorum quam algebram vocant*, Paris, Cavellat, 1560, p. 8

As for the Latin Algebra, Peletier selected some signs among Cavellat's fonts, but not those used for printing Scheubel's treatise. The reason why he gave up with using a specific sign for the second power is unclear. The letter q appears as a shortened form for the Latin word *quadratum*. For lack of a better one, this sign fits with the others ( $\mathbb{R}$ ,  $\mathcal{C}$ , and  $\beta$ ).

### 5. Conclusions

5.1 This case study concerning the relationships between authors and printers shows that the choice of mathematical signs by authors may have been here and there subject to trivial circumstances, depending on the equipment of printers. The few symbols used in the treatises are mere shortened forms for technical words. Among them, some were part of the standard equipment of printers; Some others were cut from usual handwritten signs and added to the types of some printers. More generally, the study shows, unsurprisingly, that 16th century mathematical symbolism often originated in the handwritten symbols in use before the development of printing.

5.2 In France the question of algebraic language is connected to the question of French spelling reform. In the middle of the century, authors who were engaged in the spelling reform had first to overcome the reluctance of printers towards new spelling. And even if the craftsman was benevolent, the authors had to overcome another obstacle: Printing a book written in "orthographe réformée" required the use of modern types. Only a few printers, and wealthy ones, were able to bear the cost of new types cut and melted for the convenience of one author to produce one or, at best, some books. As for the mathematics, authors and printers have probably confronted each other in the same way. The technical issue was similar for algebra (and for mathematics in general) and for spelling: Algebra required the use of special types. From the printer standpoint, typesetting a

mathematical book presented difficulties, especially in the case of a "new science" such as Algebra. As for the author, he had to make sure that the printer conformed to the handwritten *exemplar*. The typesetter had to use special characters and the author had to check the proofs of his book, as he had an expertise the printer could not pretend to have. It required a confidence between the craftsman and the scholar and an active collaboration: Good examples of these collaborations include Ramus and Wechel on the one hand, Peletier and Jean de Tournes on the other. In 16th century France, mathematics may have been, like spelling, a field in which authors established their independence over printers.

5.3 Considering the differences between the books examined above, one is led to the conclusion that the reasons why a treatise of algebra displays such or such symbols may depend less on the possession of types by printers, than on the will of authors. As regards spelling, printers may consider themselves to be experts. As regards algebra (and mathematics in general), authors had to set themselves as experts: Then, they had an opportunity to impose their view. Finally, Peletier's and Ramus' concern for the choice of algebraic signs indicates that they are committed to the mathematical language issue in general. To these authors, the matter is not only to choose the symbols, but also to typeset operations and formulas. In Peletier's and Ramus' treatises, this question seem to have been carefully considered and it would be worth studying them from that point of view.

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| l able 2: Synoptic table |                  |            |           |          |                       |  |  |  |  |
|--------------------------|------------------|------------|-----------|----------|-----------------------|--|--|--|--|
| Stifel                   | Scheubel         | Peletier   | Ramus     | Peletier |                       |  |  |  |  |
| Petreius                 | Cavellat         | de Tournes | Wechel    | Cavellat |                       |  |  |  |  |
| (1544)                   | (1551)           | (1554)     | (1560)    | (1560)   |                       |  |  |  |  |
| 22                       | 2 <sub>C</sub> , | ŖĿ         | 1.        | Ŗ        | x                     |  |  |  |  |
| 8                        | રુ               | Ş          | <b>q.</b> | Р        | $\chi^2$              |  |  |  |  |
| وو.                      | <u>دو</u>        | q          | C.        | æ        | $x^3$                 |  |  |  |  |
| ß.                       | ſz               | ß,         | ſ.        | ß        | <b>x</b> <sup>5</sup> |  |  |  |  |

Table 2: Synoptic table

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