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DISCOVERY, KNOWLEDGE AND RELIABLE LIMITING CONVERGENCE -THE K_ALC-PARADIGM

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ABSTRACT

From the point of view of the K_aLC-paradigm (Knowledge as Limiting Convergence) this paper has two aims. First of all it attempts to sketch some of the pertinent problems of scientific discovery and secondly, it outlines how these problems can be treated in the K_aLC -paradigm.

1. Introduction

Once Reichenbach had introduced the distinction between the context of justification and the context of discovery, the agenda for the part of the philosophy of science concerned with methodology was fixed. Since the discovery of hypotheses from evidence sequences was conceived as an exercise aided by divine insights, intuition and other impenetrable historical, sociological and psychological phenomena, methodology should be concerned with the justification or assessment of hypotheses relative to finite evidence sequences. The assessment of hypotheses could be done logically, *reliably* and "rational" methodological principles could be stated for this discipline. Hempel emphasized the discrepancy by later speaking of a *logic* of justification while retaining only a context of discovery. Then later others quoted others quoting Popper, quoting Hempel, quoting Reichenbach to disprove the possibility of a logic of discovery.

Even to this day, the dichotomy is still to some extent prevalent but weakened first by the early contributions of Pierce, Hanson *et al.* arguing for at least guidelines for discovery based on *abductive* reasoning. Then Gutting, Thagard, Lipton and philosophers of science akin have elaborated on these early fairly intuitively guided proposals. Today, abductive reasoning and inference to the best explanation[1] are parts of a prosperous industry for philosophers, computer scientists, logicians and cognitive psychologists. Much effort has gone into explicating the advocated methodological principles or beforementioned rationality postulates for these types of discovery processes and many valuable insights have been gained from this effort.

But there is also a different and more direct approach to discovery one may explore, one which has been greatly neglected by the methodological community. Inspired by computability theory, and under the somewhat unfortunate rubric of *formal learning theory*, methodologists like Gold [Gold 67] and later Osherson, Stob and Weinstein [Osherson 86] have presented significant results. Contrary to the dictum of Reichenbach, Hempel, Popper, etc. discovery may be studied *reliably* when applied to questions of language acquisition in which a child is asked to reliably *converge* to a grammar for its natural language. In brief, languages are modelled as recursive enumerable sets (or r. e. sets) and the child is conceived as a function required to converge to a correct r. e. index for a given set over all possible enumerations of the set. The approach is more direct (at least if abduction and inference to the best explanation cover the same ground) in the sense that a discovery method is simply understood to be function (rather than an enumeration and selection algorithm for picking the "best" explanation or hypotheses for some adequate understanding of best) taking finite evidence sequences as inputs and merely outputting hypotheses much in the original spirit leading Reichenbach to abandon the discovery endeavor.

Curiously enough, the first real learning theoretical result of interest to philosophy of science and methodology was Reichenbach's student, Putnam's, attempt to show that for any extrapolation algorithm based on Carnap's justificational standards for a probabilistic theory of confirmation, there exists a hypothesis that the Carnapian extrapolation algo-

[1] We consider abduction and inference to the best explanation to be two distinct issues even though many enjoy treating them as one and the same thing. [Hendricks & Faye 98]

rithm cannot learn even given every possible instance of the hypothesis [Putnam 63]. Putnam's argument did not exactly work, but the idea of criticizing categorical intuitive axioms of justified belief by showing that they interfere with the reliable performance of the inquiry method is important. It emphasizes that one should conduct "rational" scientific inquiry cautiously. Usually one initially fixes a set of intuitively justified canons, which the method should satisfy without ever worrying about whether these canons present impediments to finding the correct answer or not. Even a methodologist firmly committed to the proposal that his methodological principles are normative and yet indifferent to correctness may falter if it can be shown that the intuitively justified canons actually stand in the way of finding the correct answer when it could have been reliably found by a method violating them. When the methodological principles stand in the way of finding correctness, the principles are said to be *restrictive*.

But this great attempt of effective epistemology by Putnam was largely overlooked by philosophers and methodologists, perhaps because the philosophical community found it hard to see how this approach could be extended to the more classical problems of theory assessment and theory identification. However Kelly [Kelly 94], [Kelly 96], [Kelly et al. 96] has now provided a framework and produced results resting on formal learning theory but tuned to philosophy of science treating real issues of concern for this philosophical discipline.

The notion of *successful convergence* for discovery methods is of paramount importance in formal learning theory. What it includes is that there is a time such that for each later time the method outputs a consistent conjecture entailing some particular correct hypothesis and stays with it forever after no matter what evidence it subsequently receives. Assume that scientific inquiry methods of discovery and assessment are two types of *knowledge acquisition* methods. Then the fundamental assumption underlying our K_aLC-paradigm may be summarized in the following definition where Ξ denotes an arbitrary method of either assessment or discovery and h a hypothesis:

Definition 1 Limiting Convergence of Knowledge
Knowledge at a certain time = Ξ's convergence to a correct hypothesis *h* where
(1) there is a time, such that for each later time
(1.a) Ξ conjectures *h* after some finite evidence sequence has been read, and
(1.b) Ξ continues to conjecture *h* over all the possible future world courses in which *h* is correct.

From the definition it conversely follows that if Ξ has not converged to *h*, then Ξ does not know *h*. Observe that Ξ is free to vacillate any number of times prior to the modulus of convergence which cannot be specified in advance - this is limiting convergence.

Our K_aLC -paradigm includes a formal framework allowing one to investigate a multiplicity of issues among them some pertinent to theory discovery and identification:

- Definition and application of reliable discovery methods.
- The role of background knowledge when engaged in discovery problems.[2]
- The *methodological* principles placed on discovery methods.
- The restrictive or permissive character of these methodological principles (i.e. are the methodological principles correctness-conducive principles or not).
- Definitions of knowledge based on discovery methods.
- The strength of knowledge based on discovery.
- The relation between scientific discovery and scientific assessment.

This paper outlines the K_aLC-paradigm, discusses some of the issues listed above and draws the epistemological and methodological consequences important to discovery through a review of some of the results

[2] Assuming background knowledge may make some discovery problems easier to solve as for instance in E. M. Gold's recursive function identification, where the task is to identify indices for all total recursive functions *R*, the background knowledge is taken to be some subset of *R*.

obtained so far.[3]

2. The K_aLC -paradigm

2.1 Worlds, Hypotheses and Background Knowledge

Define a data stream accordingly:

Definition 2 Data Stream
A datastream ϵ is an ω-sequence of natural numbers, i. e., $\epsilon \in \omega^\omega$.

Hence, a data stream $\epsilon = (a_1, a_2, \ldots, a_n, \ldots)$ consists of code numbers of evidence, i. e., at each stage i in inquiry, a_i is the code number of all evidence acquired at this stage.

Continue to define a *possible world.*

Definition 3 Possible Worlds
1. A possible world is a pair consisting of a data stream ϵ and a state coordinate n, i. e., (ϵ, n) such that $\epsilon \in \omega^\omega$ and $n \in \omega$.
2. The set of all possible worlds $\mathbf{W} = \{(\epsilon, n) \mid \epsilon \in \omega^\omega \text{ and } n \in \omega\}$.

Let $(\epsilon \mid n)$ denote the *handle* of (ϵ, n). Furthermore, let denote $\omega^{<\omega}$ the set of all finite initial segments of elements in ω. Now $[\epsilon \mid n]$ denotes the *fan* of all infinite data streams that extends $(\epsilon \mid n)$. Finally, let $\underline{[\epsilon \mid n]} = [\epsilon \mid n] \times \omega$ denote the world fan.

Next, methods cannot see an entire world (ϵ, n) all at once. Instead the method must make do with only seeing some finite initial segment $(\epsilon \mid n)$ of it which will increase as inquiry progresses. After the method has observed the world up until and including, say, n, the world is allowed to take any course it pleases. An ornithologist may have observed 1000 black ravens up until and including n, but the world is free to show her a white or multi-colored raven at the very next stage of inquiry for all she knows. However, the world does not take multiple courses, say τ, λ, θ

[3] Please refer to [Hendricks & Pedersen98a], [Hendricks & Pedersen98b],[Hendricks & Pedersen98c],[Hendricks 99].

for the same state coordinate; the world only takes its actual course (ϵ, n). But for all the method knows at n, the world may pick any of τ, λ, θ to be its actual data stream ϵ which the fan $[\epsilon \mid n]$ represents. From n onward all data streams τ, λ, θ or σ are all accessible world courses for the same parameter on time. Let $[\epsilon \mid n]_K$ represents the background knowledge $\boldsymbol{K}$ of possible empirical possibilities or values that the world may take according to the method. The method knows that the world will take some course, just not exactly which one.

In accordance with the above, define background knowledge accordingly:

Definition 4 Background Knowledge
Background Knowledge $[\epsilon \mid n]_K = \{[\epsilon \mid n] \times \omega) \mid \boldsymbol{K}\}$, $\boldsymbol{K} \subseteq \boldsymbol{W}$.

Next, an *empirical hypothesis* is a proposition whose correctness or incorrectness only depends upon the data stream. Therefore hypotheses will be identified with sets of possible worlds.

Definition 5 Hypotheses are sets of possible worlds
The set of all empirical hypotheses $H = \mathrm{P}(\omega^{\omega}) \times \mathrm{P}(\omega)$.

2.2 Relations of Correctness

Consider the following two distinct definitions of correctness. The first definition is the one primarily entertained here even though the latter definition later on will be shown to carry some interesting properties with respect to the different definitions of knowledge.

Definition 6 Epistemic Correctness
Hypothesis h is correct$^{\mathrm{E}}$ in world (ϵ, n) iff $[\epsilon \mid n]_K \cap h \neq \varnothing$.

Definition 7 Metaphysical Correctness
Hypothesis h is correct$^{\mathrm{M}}$ in world (ϵ, n) iff $(\epsilon, n) \in h$.

The latter definition 7 will be referred to as the *metaphysical* notion of correctness as opposed the former *epistemic* notion of correctness (definition 6). The metaphysical notion says that h is correct in world (ϵ, n) just in case the world (ϵ, n) is singled out and included in h forever after. The epistemic notion maintains that h is correct in (ϵ, n) just in case (ϵ, n) agrees with h up until and including n and from there on, the actual background knowledge $[\epsilon \mid n]_K$ must only have a non-empty intersection with h, though no actval world is be picked out.

2.3 Methods of Scientific Inquiry

Now, introduce two primitive types of inquiry methods: discovery and assessment methods. First, a discovery method conjectures hypotheses in response to the evidence seen so far. Formally a discovery method may be conceived as a function, usually denoted by δ or γ, taking finite evidence sequences as inputs and outputting empirical hypotheses.

Definition 8 Discovery Methods
A discovery method δ is a function from finite initial segments of evidence to hypotheses:

$$\delta : \omega^{<\omega} \rightarrow H.$$

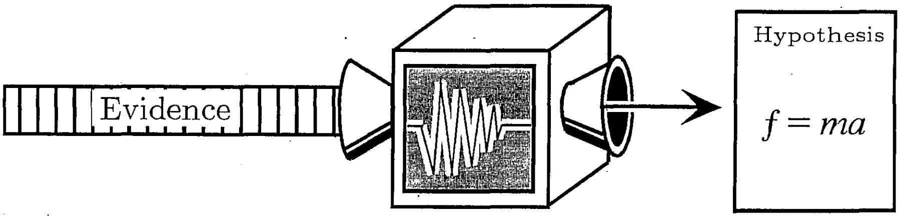

FIGURE 1: A discovery method is a function from finite initial segments of evidence to hypotheses.

The stabilization modulus (abbreviated sm_D) for a discovery method is the earliest possible time after which all the conjectures performed by the method include a consistent conjecture entailing a hypothesis h.

Definition 9 Stabilization Modulus (Discovery Methods)
$sm_D\,(\delta, h, [\epsilon \mid n]_K) = \mu k \forall n'\, n' \geq k, \forall(\mathrm{J,n'}) \in [\epsilon \mid n]_K : \delta\,(\epsilon \mid n') \subseteq h.$

That is, a method δ discovers h in $[\epsilon \mid n]_K$ if and only if $sm_D\,(\delta, h, [\epsilon \mid n]_K)$ exists.

The second inquiry primitive consists of empirical hypothesis assessment. An assessment method is a function, usually denoted by α or β taking finite evidence sequences and hypotheses as inputs and returning truth-values for the hypotheses in question.

Definition 10 Assessment Methods
An assessment method α is a function from finite initial segments of evidence and hypotheses to $\{0, 1\}$:
α: $\omega^{<\omega} \times H \rightarrow \{0, 1\}$,
where 0 denotes incorrectness and 1 denotes correctness.

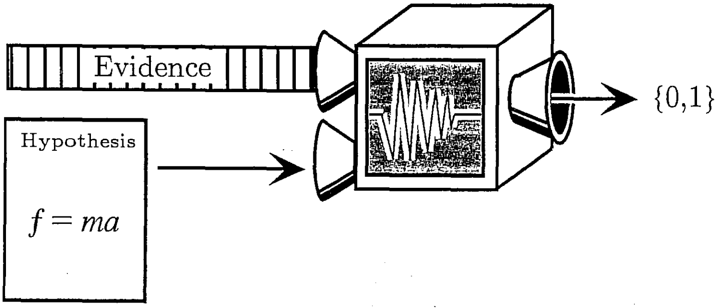

FIGURE 2: An assessment method α is a function from finite initial segments of evidence and hypotheses to $\{0,1\}$.

The assessment method may only provide an output insofar h is correct. Then the method is said to be a *verification* method since it converges to 1 if h is correct and whatever but converges to 1 if h is incorrect. If the method only provides an output insofar h is incorrect, the method is said to be a *refutation* method due to the fact that it converges to 0 if h is incorrect and anything but converges to 0 if h is correct. In case the assessment method α is a combined verification and refutation procedure, the method will be referred to as a *decision* method. Hence a decision method demands convergence to the correct value for h, whatever the correct value is:

Definition 11 Weak Limiting Decision Method α of h in (ϵ, n)
α decides h in the limit in (ϵ, n) iff
1. [if $[\epsilon \mid n]_K \cap h \neq \emptyset$ then $\exists k \forall n' \geq k$: $\alpha(h, \epsilon \mid n') = 1$],
2. [if $[\epsilon \mid n]_K \cap h = \emptyset$ then $\exists k \forall n' \geq k$: $\alpha(h, \epsilon \mid n') = 0$].

Define the convergence modulus for a weak assessment method to be the earliest possible time after which all the conjectures performed by the method pertaining to the correct value for the hypothesis in question remain the same (abbreviated cm_A^W).

Definition 12 Weak Convergence Modulus (Assessment Methods)
$cm_A^W(\alpha,(\epsilon, n), h) = \mu k \forall n' \; n' \geq k\text{: } \alpha(h, \epsilon \mid k) = \alpha(h, \epsilon \mid n')$.

One may propose a stronger definition of the limiting decision method such that α has to succeed in all worlds in accordance with the background knowledge $[\epsilon \mid n]_K$ rather than in (ϵ, n) only. This gives rise to the strong limiting decision method:

Definition 13 Strong Limiting Decision Method α of h in $[\epsilon \mid n]_K$
α decides h in the limit in $[\epsilon \mid n]_K$ iff
1. [if $[\epsilon \mid n]_K \cap h \neq \varnothing$ then $\exists k \geq n, \forall n' \geq k, \forall (J,n') \in [\epsilon \mid n]_K$: $\alpha(h, \tau \mid n') = 1$],
2. [if $[\epsilon \mid n]_K \cap h = \varnothing$ then $\exists k \geq n, \forall n' \geq k, \forall (J,n') \in [\epsilon \mid n]_K$: $\alpha(h, \tau \mid n') = 0$].

Finally, define the strong convergence modulus for an assessment method (abbreviated cm_A^S):

Definition 14 Strong Convergence Modulus (Assessment Methods)
$cm_A^S(\alpha,[\epsilon \mid n]_K, h) = \mu k \geq n, \forall n' \geq k, \forall (J,n') \in [\epsilon \mid n]_K : \alpha(\epsilon \mid k) = \alpha(\tau \mid n')$.

2.4 Inducing Assessment from Discovery

We promised to illuminate the relation between assessment and discovery and we propose that the claimed distinction between assessment and discovery is gratuitous.

Initially suppose one adopts only discovery methods as primitives. Then even when discovery methods are the only primitives it is still possible to define assessment methods. It turns out, in the most general case that for every discovery method δ one can induce an assessment method α in the following way.

Proposition 15 Discovery and Strong Assessment
If discovery method δ discovers h in $[\epsilon \mid n]_K$ in the limit, then there exists a strong limiting assessment method α which verifies h in $[\epsilon \mid n]_K$ in the limit.

The proof may be found in [Hendricks & Pedersen 98b] and is constructed by letting the strong limiting assessment method watch the discovery methods behavior and react accordingly: Assume that δ discovers h in $[\epsilon \mid n]_K$ in the limit and let $sm_D(\delta, h, [\epsilon \mid n]_K)$ be its stabilization modulus. Define α in the following way:

$$\alpha(h, (\epsilon \mid n)) = 1 \text{ iff } \delta(\epsilon \mid n) \subseteq h.$$

It is clear that if $n' \geq sm_D(\delta, h, [\epsilon \mid n]_K)$ then for all (J,n') $\in [\epsilon \mid n']_K$: $\delta(\tau \mid n) \subseteq h$. Consequently $\alpha(h, (\tau \mid n')) = 1$ and therefore $cm_A^s(\alpha, [\epsilon \mid n]_K, h) = sm_D(\delta, h, [\epsilon \mid n]_K)$. If the strong limiting assessment method α is constructed in this way, say that assessment method α is *induced* by discovery method δ.

One may induce a weak limiting assessment method in much the same way.

Proposition 16 Discovery and Weak Assessment
If discovery method δ discovers h in $[\epsilon \mid n]_K$ in the limit, then there exists a weak limiting assessment method α which verifies h in (ϵ, n) in the limit.

Inducing inquiry methods from discovery primitives will prove important later when discovery is further related to assessment in the limit of scientific inquiry in terms of knowledge transmissibility.

2.5 Defining Knowledge

2.5.1 Non-restrictive Methodological Principles

Certain non-restrictive methodological requirements will be placed on methods for discovery and assessment such that no method will be allowed to arbitrarily discover or assess hypotheses relative to finite evidence sequences. These requirements will be referred to as the *epistemic* *soundness* criteria of the methods. It turns out that the criteria of epistemic soundness are of paramount importance. They make the methods epistemically stronger than they would have been if they did not comply with these principles. Hence these principles are non-restrictive at least insofar the methods are not supposed to be effective.

Hence observe, that the following criterion of epistemic soundness is assumed for all discovery engines.

Definition 17 Epistemic Soundness$_D$ (Discovery)
Discovery method δ is epistemically sound$_D$ iff
1. If $(\mu,m) \in \delta(\tau \mid n)$ then $(\mu \mid n) = (\tau \mid n)$,
2. $\delta(\tau \mid n) \neq \emptyset$.

Hence, the hypothesis conjectured by discovery method δ must be consistent with the available evidence at n and the method is not allowed to conjecture absurdities.

Assessment methods also obey a criterion of epistemic soundness but since emphasize is on discovery it is omitted here. Please refer to [Hendricks & Pedersen 98b].

2.5.2 Definitions of Knowledge

Now, consider the following two definitions of knowledge based first on empirical hypothesis discovery (investigated in [Hendricks & Pedersen 98a]):

Definition 18 Stable Correct Belief (ST)
$K_\delta^{ST}h$ is correct in (ϵ, n) iff
1. $[\epsilon \mid n]_K \cap h \neq \emptyset$
2. $\forall n' \geq n, \forall(\tau,n') \in [\epsilon \mid n]_K$:
   2.a $\delta(\tau \mid n') \subseteq h$.

Hence δ knows h in world (ϵ, n) just in case δ truly conjectures h along ϵ at n and no matter what δ sees later, δ continues to produce a consistent conjecture entailing h. This very feature gives rise to its name: stable correct belief.

Definition 19 Strong Knowledge (SK)
$K_\delta^{SK} h$ is correct in (ϵ, n) iff
1. $[\epsilon \mid n]_K \cap h \neq \varnothing$,
2. $\forall n' \geq n, \forall (\tau, n') \in [\epsilon \mid n]_K$:
   2.a $\delta(\tau \mid n') \subseteq h$,
   2.b $(\tau, n') \in \delta(\tau \mid n')$.

This latter definition implies entailment of h by the evidence and the background knowledge, which indeed is a strong definition of knowledge. Hence if h is correct, δ eventually discovers it, and if h is incorrect, δ doesn't conjecture it - δ is *reliable*. The correct answer is forced by the data and the background knowledge.

In accordance with limiting assessment, next define *AST*-knowledge accordingly:

Definition 20 Actually Stable True Belief (AST)
$K_\alpha^{AST} h$ is correct in (ϵ, n) iff
1. $[\epsilon \mid n]_K \cap h \neq \varnothing$,
2. α decides h in the limit in (ϵ, n).

Observe that AST-knowledge is not reliably derived. We require that for the assessment method to be reliable it has to succeed unambiguously in all worlds in accordance with the background knowledge and hence adopt the strong limiting decision procedure while defining *RISK*-knowledge:

Definition 21 Reliably Inferred Stable True Belief in *K* (RISK)
$K_\alpha^{RISK} h$ is correct in (ϵ, n) iff
1. $[\epsilon \mid n]_K \cap h \neq \varnothing$,
2. α decides h in the limit in (ϵ, n).

2.6 The Formalization

This set-theoretical framework is formalizable in a modal propositional logic. The formal set-up is fully described in [Hendricks & Pedersen 98a]. In brief however, let the modal propositional language **L** consists of:

- An infinite supply of propositional letters $a, b, c, \ldots$, and brackets: (,

).

- The boolean operators $\neg$, $\wedge$, $\vee$, $\Rightarrow$ and $\Leftrightarrow$.
- A unary modal operator K^{x}_{Ξ} (read: method Ξ knows that...) for an arbitrary inquiry method of either assessment or discovery, for $x \in \{ST, SK, AST, RISK\}$ and for which the dual operator is defined as $\neg K^{x}_{\Xi}\neg$.
- Defining well-formed (wff) formulae follow standard procedure in accordance with the boolean operators, the unary operator and the closure condition.
- Transformation rules include (1) uniform substitution, (2) modus ponens, (3) the rule of necessitation.

Semantically, a model **M** is defined with respect to the set of possible worlds such that $\mathbf{M} = \langle \mathbf{W}, \varphi, \Xi \rangle$ where the denotation function φ maps proposition letters into the powerset of $\mathbf{W}$; $\varphi\ (a) \subseteq \mathbf{W}$. So proposition letters denote hypotheses and, by recursion, all wffs will denote hypotheses. Defining truth-conditions for the boolean operators follow the usual recursive recipe. For the unary operator define for example *AST*-knowledge accordingly:

Definition 22 Truth-conditions for $K_{\alpha}^{AST}A$.
$\varphi_{\mathbf{M},\,(\epsilon,\,n)}[K_{\alpha}^{AST}A] = 1$ iff
3. $[\epsilon \mid n]_{K} \cap [A]_{\mathbf{M}} \neq \varnothing$,
4. α decides $[A]_{\mathbf{M}}$ in the limit in (ϵ, n).

Truth-conditions for *RISK, ST* and *SK* knowledge are defined in the same way furnishing the obvious modifications.

3. Significant Epistemological Results

Let Ξ be an arbitrary inquiry method of either assessment or discovery. Then the modal system S4 consists of the following axioms.

i. $K_{\Xi}A \Rightarrow A$. **AXIOM OF TRUTH (*T*)**. *Interpretation. If method* j *knows A, then A is true.*
ii. $K_{\Xi}(A \Rightarrow C) \Rightarrow (K_{\Xi}A \Rightarrow K_{\Xi}C)$. **DEDUCTIVE COGENCY AXIOM (*K*)**. Interpretation: If the method correctly converges to $A \Rightarrow C$, then if

the method also correctly converges to A, the method correctly converges to C.

iii. $K_{\Xi}A$ Í $K_{\Xi}K_{\Xi}A$. **SELF-AWARENESS AXIOM (or the KK-THESIS) (*4*)**. Interpretation: The method knows the data streams on which it correctly converges. So if the method correctly converges to A, the method correctly converges to the belief that the method is converging.

3.1 Soundness

The following four propositions regarding knowledge have already been proved and reveals something about the strengths of the different definitions of knowledge entertained.

Proposition 23 ST, SK, AST, RISK and S4. [Hendricks & Pedersen 98a, 98b]
If knowledge is defined as either ST, SK, AST or RISK, then knowledge validates S4.

3.2 A Negative Result with a Positive Outcome

Suppose knowledge is defined as strong knowledge in accordance with definition 19 and its obvious modal counterpart. Then the strongest modal system S5 consists of the Axiom of Truth, Deductive Cogency Axiom and the following axiom (5):

$$\neg K_{\Xi}A \Rightarrow K_{\Xi}\neg K_{\Xi}A,$$

referred to as the Axiom of Wisdom. Now the following property holds:

Proposition 24 SK and S4. [Hendricks & Pedersen 98a]
If knowledge is defined as SK, then knowledge cannot validate the Axiom of Wisdom and consequently neither satisfy S5.

Some significant epistemological consequences follow from proposition 24. Indeed some skeptics have argued that if knowledge is anything worth pursuing at all, then it should be of the metaphysical reality. Then again they argue that such an endeavor is futile from the outset. According to the Academic skeptics, the only thing they know is that they do not know. For the Academic skeptics this axiom holds both for all possible convergence criteria. But insofar it holds a finite time convergence criterion the Pyrrhonian skeptic Sextus Empiricus launched the classical diagonal argument against inductive inference and hereby attacked the Academic skeptics by concluding that their position was just as dogmatic as Sextus took Plato's conception of knowledge to be. But suppose the Axiom of Wisdom only is designed to hold in the limiting case. Then the Academics could in principle argue that they *always* are in the advantageous position of possessing a *reliable* method enabling them to converge to the fact that they do not know. However proposition 24 reveals that if knowledge as convergence is taken seriously, then if convergence has not arised we simply do not know. The essential reason is that there is no way to control δ's behavior prior to the modulus of convergence. We cannot know that we do not even for the strongest possible conceptions of knowledge and correctness even in the limiting case. And this goes whether we insist on being ambitious epistemologists repelling attacks from knowledge skepticism ala the Academics or insist on being Academics ourselves. In sum, one cannot converge to ones ignorance - Sextus said so in finite time and proposition 24 says so in the limit.

3.3 Knowledge Transmissibility

Final clarification of the relations between assessment and discovery in the limit of scientific inquiry may be accomplished by studying the conditions of knowledge transmissibility. Under what circumstances may one method transfer its knowledge of a hypothesis to another method when this latter method is equipped only with knowledge of the former method's knowledge of some particular hypothesis in question.

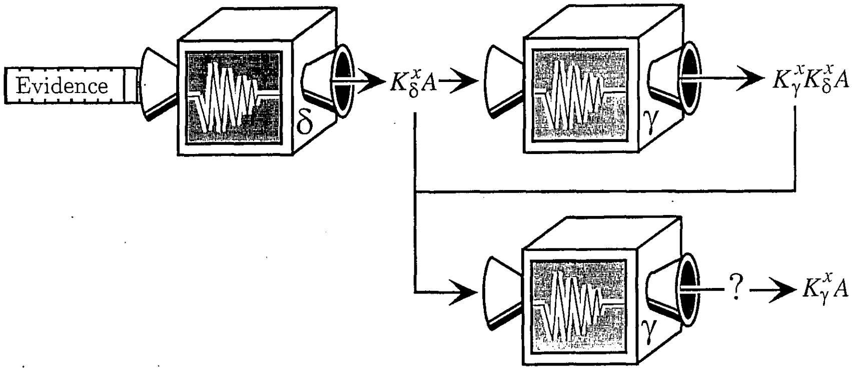

FIGURE 3: Knowledge transmissibility for discovery methods

Knowledge transmissibility was first studied by Hintikka in [Hintikka 62] by considering whether

$$K_a K_b p \Rightarrow K_a p$$

held for his definition of knowledge where a,b denote agents or methods and p denotes some arbitrary proposition. But in its simple form $K_a K_b p \Rightarrow K_a p$ will not immediately work here in the limiting convergence paradigm since we have multiple methods and multiple definitions of knowledge. Hence care has to be taken to respect these additional constraints. For instance, not only is it possible that a method δ having, say, ST-knowledge of the fact that another γ has SK-knowledge of some hypothesis A, may obtain ST-knowledge of this hypothesis A. But may δ even obtain strong knowledge of the hypothesis that γ has strong knowledge of and insofar bargain its way to convert its ST-knowledge to SK-knowledge? Furthermore, since discovery methods can induce assessment methods under certain circumstances does it follow that δ may, by observing assessment method α's behavior, change its knowledge type from one based on discovery to one based on assessment, i. e., does $K_\delta^{ST} K_\alpha^{RISK} A \Rightarrow K_\delta^{RISK} A$ for instance hold? In general we consider whether

$$K_\Theta^{\ y}\ K_\Xi^{\ x} A \Rightarrow K_\Theta^{\ z} A$$

is valid for all possible *legitimate* combinations of methods and "knowledges". Call a transmissibility instance legitimate if it results from not confusing types of knowledge based on assessment methods (α, β) with types of knowledge based on discovery methods (δ, γ). For instance

$$K_\alpha^{ST} K_\gamma^{RISK} A \Rightarrow K_\alpha^{RISK} A$$

constitutes an illegitimate transmissibility instance since an assessment method α cannot be accredited with ST- nor SK-knowledge directly as little as a discovery method γ may be accredited with RIST-knowledge directly - *only through possible discovery inducement*. Formally respecting this additional feature of inducement requires supplying the modal system S4 with what we have called multiple method extensions *MMS* (See additionally [Hendricks & Pedersen 98b].

3.3.1 Uniform Transmissibility

Call knowledge transmissibility *uniform* if it results from two different methods of the same inquiry type (either discovery or assessment exclusively) holding the same kind of knowledge. Then the following theorem holds:

Theorem 25 Uniform Transmissibility [Hendricks & Pedersen 98b]

Method		$K_\Theta^y K_\Xi^x A$		$K_\Theta^z A$
Discovery	1.	$x,y \in \{ST\}$	$z \in \{ST\}$	$K_\delta^{ST} A$
$\Theta \in \{\delta\}$, $\Xi \in \{\gamma\}$	2.	$x,y \in \{SK\}$	$z \in \{SK\}$	$K_\delta^{SK} A$
Assessment	3.	$x,y \in \{AST\}$	$z \in \{AST\}$	$K_\alpha^{AST} A$
$\Theta \in \{\alpha\}$, $\Xi \in \{\beta\}$	4.	$x,y \in \{RISK\}$	$z \in \{RISK\}$	$K_\alpha^{RISK} A$

3.3.2 Semi-uniform Transmissibility

Call knowledge transmissibility *semi-uniform* if it results from two different methods of the same inquiry type (either discovery or assessment exclusively) holding different kinds of knowledge based either on discovery or assessment exclusively. Then the following theorem holds:

Theorem 26 Semi-uniform Transmissibility [Hendricks & Pedersen 98b]

Method	$K_\Theta{}^y K_\Xi{}^x A$			$K_\Theta{}^z A$
Discovery	1.	$x \in \{\text{SK}\}$, $y \in \{\text{ST}\}$	$z \in \{\text{ST}\}$	$K_\delta{}^{ST} A$
$\Theta \in \{\delta\}$, $\Xi \in \{\gamma\}$	2.	$x \in \{\text{SK}\}$, $y \in \{\text{ST}\}$	$z \in \{\text{SK}\}$	-
	3.	$x \in \{\text{ST}\}$, $y \in \{\text{SK}\}$	$z \in \{\text{SK}\}$	$K_\delta{}^{SK} A$
	4.	$x \in \{\text{ST}\}$, $y \in \{\text{SK}\}$	$z \in \{\text{ST}\}$	$K_\delta{}^{ST} A$
Assessment				
$\Theta \in \{\alpha\}$, $\Xi \in \{\beta\}$	5.	$x \in \{\text{RISK}\}$, $y \in \{\text{AST}\}$	$z \in \{\text{AST}\}$	$K_\alpha{}^{AST} A$
	6.	$x \in \{\text{RISK}\}$, $y \in \{\text{AST}\}$	$z \in \{\text{RISK}\}$	-
	7.	$x \in \{\text{AST}\}$, $y \in \{\text{RISK}\}$	$z \in \{\text{RISK}\}$	$K_\alpha{}^{RISK} A$
	8.	$x \in \{\text{AST}\}$, $y \in \{\text{RISK}\}$	$z \in \{\text{AST}\}$	$K_\alpha{}^{AST} A$

3.3.3 Non-uniform Transmissibility

Finally, call knowledge transmissibility *non-uniform* (or mixed) if it results from two different methods of different inquiry type (either discovery or assessment) holding different kinds of knowledge based either on discovery or assessment. Then the following theorem holds:

Theorem 27 Uniform Transmissibility [Hendricks & Pedersen 98b]

Method		$K_\Theta{}^y\, K_\Xi{}^x A$		$K_\Theta{}^z A$
$\Theta \in \{\alpha\}$	1.	$x \in$ {AST}, $y \in$ {NST}	$z \in$ {AST}	$K_\delta{}^{AST} A$
$\Xi \in \{\delta\}$	2.	$x \in$ {AST}, $y \in$ {NST}	$z \in$ {AST}	-
$\Theta \in \{\delta\}$	3.	$x \in$ {AST}, $y \in$ {SK}	$z \in$ {SK}	$K_\delta{}^{SK} A$
$\Xi \in \{\alpha\}$	4.	$x \in$ {AST}, $y \in$ {SK}	$z \in$ {AST}	$K_\delta{}^{AST} A$
$\Theta \in \{\alpha\}$	5.	$x \in$ {ST}, $y \in$ {AST}	$z \in$ {AST}	$K_\delta{}^{AST} A$
$\Xi \in \{\delta\}$	6	$x \in$ {ST}, $y \in$ {AST}	$z \in$ {ST}	-
$\Theta \in \{\alpha\}$	7.	$x \in$ {SK}, $y \in$ {AST}	$z \in$ {AST}	$K_\delta{}^{AST} A$
$\Xi \in \{\delta\}$	8.	$x \in$ {SK}, $y \in$ {AST}	$z \in$ {SK}	-
$\Theta \in \{\alpha\}$	9.	$x \in$ {SK}, $y \in$ {RISK}	$z \in$ {RISK}	$K_\delta{}^{RISK} A$
$\Xi \in \{\delta\}$	10.	$x \in$ {SK}, $y \in$ {RISK}	$z \in$ {SK}	-
$\Theta \in \{\alpha\}$	11.	$x \in$ {ST}, $y \in$ {RISK}	$z \in$ {ST}	$K_\delta{}^{ST} A$
$\Xi \in \{\delta\}$	12.	$x \in$ {ST}, $y \in$ {RISK}	$z \in$ {ST}	$K_\delta{}^{ST} A$

The classical dichotomy emphasizing the difference between assessment and discovery is simply false from the limiting convergence point of view. Discovery methods can induce assessment methods - *even reliable ones*. The classical dichotomy conversely claims that only assessment methods may provide reliable means for getting to the correct answer. But it has been shown that even unreliable discovery methods, (i. e,.ST-knowledge engines) may induce reliable assessment methods both in the weak and in the strong sense. Finally, knowledge transmissibility taught us how assessment and discovery may converge in the limit of empirical scientific inquiry so *in the (limiting) end there is not much of a difference between assessment and discovery* (Theorem 27).

4. Conclusion

We have attempted to show how some pertinent philosophical and methodological issues related to scientific discovery may be adequately dealt with in the K_aLC-paradigm.

Some may object, first of all, that the model of scientific discovery presented is oversimplified. The model does not take into account that much theory identification includes the formation of new theoretical concepts to describe the (in)-discrete observations properly. True, but that only goes to show that their is a substantial difference between inductive discovery, discussed currently, and abductive discovery. Abductive is genuinely more complicated than inductive discovery because it requires a leap out of the vocabulary of the evidence language forming new theoretical concepts in the hypothesis language.

Secondly, some may correctly point out that inference to the best explanation (IBE) is not dealt with in any significant way either. Again, there is also a difference between IBE and inductive discovery. IBE consists of an enumeration and selection algorithm for choosing between competing hypotheses in case truth is underdetermined. But then IBE is a sort of assessment method rather than a discovery method. The hypotheses subject to enumeration by an IBE-method may be either abductively or inductively related to the evidence language. The challenge for IBE-methodology is to provide some adequate measure of ''best'' and attempts have been made to cash it out by following methodological principles like simplicity, coherence, unification, consilience, entrencement, consistency to mention but a few. However observe that if a methodologist favors *myopic* inquiry (gaining truths and avoiding errors) as Levi calls it (as opposed to categorical methodology or *messianic* inquiry in which the norms are motivated, not hypothetically, as means for finding the truth, but categorically as ends in themselves) she has to ensure that the methodological principles advocated, for a proper IBE-algorithm, are truth-conducive, non-restrictive and reliable means for getting to the truth.

Finally we point to some further applications of the K_aLC -paradigm. Note initially that the definitions of knowledge presented here by no means are the only entertainable ones - the list of possible definitions is inexhaustible. Second it is possible to define both belief- and certainty-operators for which their respective strengths may be subject to investigation. Additionally one may also choose to consider the interrelations between knowledge, belief and certainty through conditions of general epistemic transmissibility. Note that the hypotheses under investigation have been primitively identified with the set of possible worlds in which they are true but since the K_aLC -paradigm automatically features a branching time model of possible worlds, modal learning may also be dependent upon, not only the methodological recommendations, but also the temporal complexity of the hypotheses [Hendricks & Pedersen98d]. Finally, we now know that the entertained definitions of knowledge validate the axioms listed, i. e., soundness but we do not know yet whether or not these axioms are exhaustive for the respective definitions. Proofs of completeness reveal such facts. All these further investigations are to be found in [Hendricks 99].

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